A Drift Diffusion Model of Proactive and Reactive Control in a Context-Dependent Two-Alternative Forced Choice Task

Olga Lositsky  
Princeton Neuroscience Institute  
Princeton University  
lositsky@princeton.edu

Robert C. Wilson  
Department of Psychology  
University of Arizona  
bob@email.arizona.edu

Michael Shvartsman  
Princeton Neuroscience Institute  
Princeton University  
ms44@princeton.edu

Jonathan D. Cohen  
Princeton Neuroscience Institute  
Princeton University  
jdc.princeton.edu

Abstract

Most of our everyday decisions rely crucially on context: foraging for food in the fridge may be appropriate at home, but not at someone else’s house. Yet the mechanism by which context modulates how we respond to stimuli remains a topic of intense investigation. In order to isolate such decisions experimentally, investigators have employed simple context-based decision-making tasks like the AX-Continuous Performance Test (AX-CPT). In this task, the correct response to a probe stimulus depends on a cue stimulus that appeared several seconds earlier. It has been proposed (Braver, 2007) that humans might employ two strategies to perform this task: one in which rule information is proactively maintained in working memory, and another one in which rule information is retrieved reactively at the time of probe onset. While this framework has inspired considerable investigation, it has not yet been committed to a formal model. Such a model would be valuable for testing quantitative predictions about the influence of proactive and reactive strategies on choice and reaction time behavior. To this end, we have built a drift diffusion model of behavior on the AX-CPT, in which evidence accumulation about a stimulus is modulated by context. We implemented proactive and reactive strategies as two distinct models: in the proactive variant, perception of the probe is modulated by the remembered cue; in the reactive variant, retrieval of the cue from memory is modulated by the perceived probe. Fitting these models to data shows that, counter-intuitively, behavior taken as a signature of reactive control is better fit by the proactive variant of the model, while proactive profiles of behavior are surprisingly better fit by the reactive variant. We offer possible interpretations of this result, and use simulations to suggest experimental manipulations for which the two models make divergent predictions.

Keywords:  cognitive control; context-dependent decision-making; proactive and reactive control; dual-mechanisms of control; drift diffusion model

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1 Introduction

Most of our daily decisions depend crucially on context. While our normal response to seeing a new email from a good friend might be to open it, this response may be suppressed if our goal is to work. However, if an email alert were to arrive about an approaching winter storm, we should be able to respond to it despite its former irrelevance. This ability to modulate our behavior based on context has been termed “cognitive control.” The traditional account of these context effects is a proactive one, in which context is represented by patterns of ongoing neural activity and modulates the response to incoming stimuli (Miller & Cohen, 2001). The Dual Mechanisms of Control (DMC) framework is a recent proposal is that active task preparation and maintenance is costly, and the brain may sometimes use a reactive strategy, where context is stored passively (perhaps in the pattern of synaptic weights) and reactivated when the relevant stimulus is presented (Braver, 2007). Under the DMC, the choice between strategies is based on intrinsic motivation, the constraints of cognitive architecture, and the particular demands of the task. While the DMC framework has inspired a large body of experimental work in humans, we provide the first formal computational instantiation of it, and apply it to the AX-Continuous Performance Test (AX-CPT), a classic task from the cognitive control literature. In the AX-CPT, a context cue (A or B) is presented briefly on each trial, followed by a probe stimulus (X or Y) after a short delay. Participants must respond to the probe stimulus differently depending on whether it was preceded by (i.e. in the context of) an A or a B.

We model proactive control as identifying the probe (X or Y) in the context of the cue (A or B), and reactive control as recalling the cue (A or B) in the context of the probe (X or Y). We instantiate the probe identification or cue recall processes as drift-diffusion models, which allow us to predict the pattern of choices and reaction times for the different trial types in the task. Using these models, we show that choice behavior taken as a signature of proactive or reactive control can be produced by both instantiations of our model, though they make different predictions for reaction time distributions. Our simulations suggest that manipulating uncertainty about the cue and probe separately could bring subjects into a parameter regime where the two models make very different predictions. Moreover, we find – counter-intuitively – that proactive behavior is better fit by the model we termed “reactive”, while reactive behavior is better fit by the model we termed “proactive”, and offer possible interpretations.

2 Methods

2.1 The AX-CPT Task

On each trial of the task, a cue (A or B) signals how to respond to the upcoming probe (X or Y), which appears a few seconds later. In the variant we use, the subject must press the left button on AX and BY trials, and the right button on AY and BX trials (Fig. 1). Manipulating the conditional probability of cue and probe stimuli can be used to investigate subjects’ control strategy. A subject proactively preparing responses will be biased to respond incorrectly on AY trials, but not BX trials (because AX is far more likely conditioned on A, and BX is more likely conditioned on B). On the other hand, a subject reacting to the probe will be biased to respond incorrectly on BX trials, but not AY trials (because AX is far more likely conditioned on X, but AY is more likely conditioned on Y). The relative proportion of AY and BX response times and errors in such designs are therefore taken to reflect an index of proactive or reactive control (Braver, 2009).

2.2 Behavioral Experiments to Induce Proactive and Reactive Strategies

To encourage subjects to use both types of control, we used a number of within-subject manipulations. To induce a proactive bias, participants were rewarded for responding correctly faster than 500ms: we expected participants to choose proactive processing of context to enable faster responding. To induce a reactive bias, we interleaved the AX-CPT task with a competing task, making it more difficult to prepare and actively maintain the context.

We ran 11 small experiments, varying trial frequencies (while maintaining equal AY and BX joint probabilities), the duration of the delay period, and the difficulty of the second task. While these variations are interesting and will be reported separately, here we aggregate the results from all experiments (120 subjects) in order to draw general inferences about the proactive-bias and reactive-bias conditions.
2.3 Drift Diffusion Model of Context-Dependent Decision-Making on the AX-CPT

The AX-CPT is fundamentally a two-alternative forced choice (2AFC) task: participants make a decision between left and right on the basis of a context cue (A or B) and a probe stimulus (X or Y). Such 2AFC tasks have been modeled successfully using drift diffusion models (DDMs), in which a decision is made by gradually accumulating evidence for either left or right response and responding when the evidence crosses some decision threshold, \( z \) (see Bogacz, 2006, for a detailed review). Unlike context-free 2AFC tasks, in the AX-CPT, subjects must integrate memory of the cue they saw seconds earlier with incoming perceptual evidence about the probe, and the dynamics of this integration should vary by strategy. We implemented the proactive strategy as a drift diffusion process reflecting perceptual evidence about the probe, the parameters for which are set by the cue noise encoded in memory. We modeled the reactive strategy as a drift diffusion process reflecting retrieval of the cue from memory, the parameters of which are set by noisy perception of the probe. This distinction reflects the DMC conception of proactive control as task configuration triggered by the cue and reactive control as task configuration triggered by the probe.

2.3.1 Drift Diffusion-on-Probe (DDP) model of “proactive” strategy

In the proactive variant, which we will term “Drift Diffusion-on-Probe” (DDP), the true cue, \( C \), is encoded in memory as \( \bar{C} \), with some bit-flip noise, \( \epsilon_C \), which reflects the probability that the cue is encoded incorrectly. Given its noisy cue representation, the model computes the posterior probability distribution over the rule, \( R \) (i.e. \( R = \) “left on \( X \) right on \( Y \)” or “right on \( X \), left on \( Y \)” ) using Bayesian inference: \( P(R|\bar{C}) \propto P(\bar{C}|R)P(R) \), with the prior set based on trial frequencies, and the likelihood computed using the true value of \( \epsilon_C \). The model then samples the rule from \( p(R|\bar{C}) \), and uses this sampled rule to set up the parameters of the DDM on the probe. The selected rule determines the direction of the drift, the threshold, \( z \), and starting point, \( x_0 \), for drift diffusion on the probe. The model therefore has two starting point and threshold parameters (one for each cue: \( \bar{x}^A, \bar{x}^B, x_0^A, x_0^B \) ), a drift rate parameter for the probe (\( d \) ), cue noise (\( \epsilon_C \) ) and a response time offset due to motor planning and execution (T0) – seven parameters in total.

These assumptions, in combination with standard results from the DDM literature (Busemeyer & Townsend, 1993; Navarro & Fuss, 2009), allow us to compute reaction time distributions for correct and error trials in each of the four trial types (AX, AY, BX, BY). This, in turn, allows us to fit the model to the experimental data.

To develop an intuition about how the model behaves, consider that the probability of using Rule B given that \( \bar{C} = \bar{B} \) increases with the frequency of B cues and decreases with encoding noise, \( \epsilon_C \). As \( \epsilon_C \) increases, the model will be more likely to confuse B for A cues, given the higher frequency of A. This will cause it to respond incorrectly on BX, and produce the typically “reactive” mistake. In other words, the higher the cue noise, the more reactively our “proactive” model behaves.

2.3.2 Drift Diffusion-on-Cue (DDC) model of “reactive” strategy

For the reactive variant, which we term “Drift Diffusion-on-Cue” (DDC), we reversed this process to reflect the fact that the reactivation of task representations is triggered and modulated by the probe. In this variant, the probe is encoded with some bit-flip noise \( \epsilon_P \), and must be decoded in order to set the rule (e.g. Rule = “Right on A, Left on B”). Again, as the probe encoding noise increases, the model will be more likely to confuse \( Y \)’s for \( X \)’s, given the higher frequency of \( X \)’s, causing it to respond incorrectly on AY trials. By a similar intuition as above, our “reactive” model will produce typically “proactive” mistakes as probe noise increases. The sampled rule also sets the starting point for drift diffusion on the cue. Given the trial frequencies (Fig. 1), a rational subject might set the X starting point closer to the Left, but the Y starting point closer to the Right threshold.

Note that both variants of the model implement noise in the cue and probe representations, but they do so in different ways. Since we have no direct measure of the processing during the delay period, both models try to predict the time between probe onset and the subject’s response. The DDP model assumes that the cue has been decoded during the delay period, such that probe processing is the time-limiting step in the
decision. The DDC model considers the probe to be already processed (e.g. during T0), and uses the noisy probe representation to modulate the retrieval of the cue, which is what determines the variability in response times.

2.4 Fitting our Model to Data

We used maximum likelihood estimation to fit both the DDP and DDC models separately for each subject in each experimental condition. To compute the likelihood of each trial given the model, we used an approximation to the probability density function of a Wiener diffusion process of which the DDM is one example (Navarro & Fuss, 2009). We also fit two benchmark diffusion models: a homogeneous model and a saturated model that should reflect the most and least constrained forms of our models. Since our DDP and DDC models produce a mixture of two DDM reaction time distributions (by sampling from two contextual rules), we also implemented the benchmark models as weighted sums of two DDMs. The homogeneous model assumes no variation by trial type and assumes all trials are drawn from a mixture of two arbitrary DDMs. The saturated model assumes no shared properties between trial types and assumes each trial type is drawn from a separate mixture of two arbitrary DDMs. The resulting saturated model has 32 parameters, and thus many more degrees of freedom than our 7-parameter models. We use these additional models to compute a pseudo-$R^2$ statistic that situates our theoretically-informed model in the space of possible Wiener process models. We follow Fernandes (2014) to define this statistic as follows:

$$pR^2 \equiv 1 - \frac{L_S - L_T}{L_S - L_H}$$

where $L_S$ is the log likelihood of the saturated model, $L_T$ is the log likelihood of the theoretically-informed model, and $L_H$ is that of the homogeneous or null model. This measure can be interpreted as the relative reduction in deviance due to additional covariates. A value of 0 means our theoretical model does no better than a model that does not distinguish between trial types, and a value of 1 means that our model performs as well as a model that fits each trial type separately.

2.5 Landscaping Analysis: Finding where the models make distinct predictions

To infer in which part of the parameter space the DDP and DDC models make the most distinct predictions, we performed a landscaping analysis (Navarro, Pitt & Myung, 2004): we sampled behaviors uniformly from one model, and fit this behavior with both models (repeating with both models as the simulating model). This analysis lets us identify parts of the parameter space where one model cannot fit the other’s generated data, and therefore the models make distinct predictions.

3 Results

3.1 Behavioral Results

As Figure 3 shows, the proactive manipulations generally succeeded at eliciting more AY than BX errors ($p=0.001$), while the reactive manipulations were more variable at eliciting more BX than AY errors ($p=0.06$). However, unlike studies that did not use RT deadlines (Braver, 2009), mean reaction times showed no consistent difference between the two conditions. It is likely that incentivizing subjects to respond faster than 0.5s reduced the variability in decision times.

[Figure 3 Summary of behavior in the two experimental conditions.]

3.2 Model Goodness of Fit by Experimental Condition and Model Type

For both DDP and DDC, fit to both experimental conditions, the mean pseudo $R^2$ ranged between 0.65 and 0.73 (Fig.4A). Since the saturated model had four times as many parameters, our models received a consistently lower BIC than the saturated model (all $p<10^{-16}$), suggesting that our models are a parsimonious description of the data. We discuss more detailed results next.
3.2.1 DDM on Cue Model Fits Proactive Behavior Better; DDM on Probe Model Fits Reactive Behavior Better

Within the proactive condition, the DDC model fit behavior significantly better than DDP ($p<10^{-3}$) while there was no significant difference between models for the reactive condition ($p=0.94$). However, since there were large individual differences in how well the manipulation elicited the desired behavior, we also split subjects into two groups based on their proactive error index (AY errors – BX errors.) This revealed that proactive behavior was significantly better fit by the DDC than the DDP model ($t(110)=10.00; p<10^{-17}$) while reactive behavior was better fit by the DDP than the DDC model ($t(106)=3.64; p<10^{-4}$, Fig.4B). This pattern was the opposite of what we had predicted. One interpretation is that, since we found significant differences across conditions in the proactive error, but not the proactive RT index, AY and BX errors were not accompanied by the slower responding, which a lower drift rate on the probe or cue would predict, such that variation in encoding noise could better capture these error profiles. The DDC model can produce higher BX errors (withouth the accompanying slower RTs) by increasing $\varepsilon_p$, while the DDP model can produce higher AY errors in the same way. We are currently running experiments without an RT deadline, to see if this effects holds when we induce more natural variability in reaction time distributions. Directly modeling the effect of RT deadline within our DDMs is another direction we are pursuing.

3.2.2 Behavior in reactive condition was more noisy overall than in proactive condition

Splitting the data by experimental condition revealed that both of our models fit the proactive condition better than the reactive condition ($t(119)=2.09, p=0.04$ for DDP and $t(119)=2.73, p=0.007$ for DDC, Fig. 4A.) Given that we did not reward speed or accuracy in the reactive condition, participants were likely more unstable in their strategy across trials. Thus, such behavior might be better fit by the saturated model, since it fits a weighted sum of two arbitrary DDMs. However, splitting behavior by the proactive error index did not result in lower pseudo-$R^2$ scores for reactive behavior ($p=0.64$ for DDC), and the DDP model even trended towards fitting reactive behavior better ($p=0.05$.) Thus, while the reactive manipulation may have promoted more noisy behavior, our models can fit reactive profiles of behavior as well as proactive ones.

3.2.3 Landscaping analysis points to more sensitive experimental manipulations for model identification

Besides removing the cap on reaction times, our simulations suggest experimental manipulations that can help to test the predictions of our two models more precisely. Comparing the log likelihoods of the two models (for data generated by one of them) for various parameter combinations revealed that when the encoding noise on one stimulus is high, and the drift rate on the other stimulus is high, the two models make the most distinct predictions. To test this, we plan to parametrically degrade the cue and probe stimuli independently of each other. We predict that the DDP model will fit better when the probe uncertainty exceeds the cue uncertainty, and vice versa. In addition, we plan to test how the proactive index (AY – BX errors/RTs) relates to the time spent integrating probe or cue evidence, respectively.

References