Learn to Integrate Mathematical Models in Human Performance Modeling

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The current paper provided a tutorial of the integration of mathematical models in human performance modeling. It introduced the unique features of mathematical modeling in human performance, and the steps in mathematical model integration, including how the literature of models was reviewed, how a research gap was identified, and how a mathematical model was developed and integrated based on existing models, and how a model was validated via an experimental study. A case study was presented by following each step to illustrate the integration of several existing models to derive a new model of drivers’ braking performance in warning response with its integration with the existing mathematical models of driver speed control in normal situations and the model of humans’ warning response time. This is the first tutorial work that provided a detailed explanation of the steps in mathematical model integration with a case study in human performance modeling. It could be used as guidance for human factors professionals to learn the mathematical modeling approaches and will benefit the field of human performance modeling.

INTRODUCTION

Human performance modeling is an emerging area that focuses on the development and application of the quantitative and predictive model of human cognition and behavior. These models, including mathematical models and simulations models, provided insight into the mechanism of the various aspects of perception, cognition, and motor performance. Mathematical models take the form of equations, whereas simulation models utilize the form of computer simulations. Compared with simulation models, mathematical models have several advantages in quantifying human performance.

First and foremost, mathematical models of human behavior can be relatively easily edited, modified, improved, and integrated together to develop new mathematical equations. The integrated model can be used to quantify new components and tasks in human performance.

Secondly, mathematical models of human behavior extract the mechanisms of human behavior by clear quantifications of the relationships between the inputs and outputs of each equation. With this regards, it is easier for users to understand and extract the relationships among variables using these mathematical models than reading computer codes. The simplicity of mathematical models makes it understandable to design professionals since the models illustrate design trade-offs between different variables.

Thirdly, mathematical models of human behavior and performance can be relatively easily implemented in different programming languages and embedded in various intelligent systems to guide and improve system design.

Fourthly, mathematical models and equations can lead to analytical solutions, which is more accurate than simulator results.

Lastly, there are mathematical models and equations quantifying the entire human cognition system, which is another unique feature of the mathematical modeling approach.

This article aims to demonstrate how the mathematical model and equations can be integrated and developed to quantify and predict new aspects of human behavior and performance. Through this tutorial, the steps in model integration and development will be summarized in a general format, and a case study will then be presented as an example to illustrate how a new model can be derived via the integration of existing mathematical models in human performance modeling.

THE STEPS IN MATHEMATICAL MODEL INTEGRATION AND DEVELOPMENT

In order to quantify human performance in new phenomena and tasks, the mathematical models and equations can be developed based on the integration of exiting models to predict human behavior and performance in the new tasks. The model development included the following steps.

Step 1: Literature Review of the Research Problem and Existing Mathematical Models

In the first step, the relevant literature on the research problem should be reviewed to search for any existing models that addressed the similar problems. Most tasks and problems to be modeled could be broken down into several components that fit in different areas of human behavior and performance, such as attention and perception, cognition, action or motor control. In each area, there are existing mathematical models that can be used to quantify the similar aspects of the target research problem, such as visual search and visual sampling (Wickens, 2008), memory (Laughery, 1970), judgment and decision-making (Gao & Lee, 2006), and target pointing (Fitts, 1954). Moreover, the relevant theoretical models and empirical studies also provide insights into model development.

To be noted, a website established by the author can be utilized as a mathematical model library to search existing mathematical models of human performance with Queueing Network-Model Human Processor (QN-MHP) (“Mathematical modeling with QN-MHP”, 2012). As shown in Figure 1, QN-MHP is a computational architecture that integrates three
discrete serial stages of human information processing (i.e. perceptual, cognitive, and motor processing) into three continuous subnetworks. Each subnetwork is constructed of multiple servers and links among these servers (Wu & Liu, 2008). Each server is an abstraction of a brain area with specific functions, and links among servers represent neural pathways among functional brain areas. The neurological processing of stimuli is illustrated in the transformation of entities passing through routes in QN-MHP.

Since this architecture was established, QN-MHP has been applied to develop mathematical models to quantify various aspects of human cognition and performance. In terms of human perception, new equations have been integrated to model eye movements, and speed perception (Lim & Liu, 2009; Wu & Liu, 2008; Zhao & Wu, 2013). Regarding human cognition and decision-making, mathematical models have been developed to predict textual information chunking (Wu & Liu, 2008), dual-task interference (Lin & Wu, 2012), and complex decision-making (Zhao & Wu, 2013). In terms of human motor control, mathematical models have been built to model motor program retrieval (Wu & Liu, 2008), bimanual coordination in typing tasks (Lin & Wu, 2012; Wu & Liu, 2008), and foot movement time and amplitude (Zhao & Wu, 2013). Moreover, the characteristics of the entire human cognition system have also been quantified with models developed based on QN-MHP, such as human mental workload (Wu & Liu, 2007; Cao & Liu, 2015), and the reinforcement learning process (Wu & Liu, 2008).

Step 2: Identification of the Research Gap between Existing Models and the Target To Be Modeled

Secondly, the research gap has to be identified before the development of the new model to connect the existing modeling field and the target modeling outcomes. There are two major types of research gaps. One type of the research gap is associated with the transformation of the model from a general problem to a specific problem, or from an existing domain to the target domain of the research problem. As an example for the adaptation of a general model to a model in a particular task, the modeling of the operators’ trust to automation systems (Gao & Lee, 2006) can be converted to quantify the drivers’ trust to autonomous vehicles. As an example for the modification of an existing model to address a similar problem in a different domain, a visual search model of the detection of ground vehicles in images (Witus and Ellis, 2003) could be modified to quantify the visual search model of dangerous vehicles and pedestrians in the field of view. The other type is relevant to the missing components to model the research problem. Although human information processing in most tasks can be broken into perception, cognition, and motor control, perception-action models are more commonly developed in the modeling literature due to the complexity of the cognition component. For instance, in order to predict the
interaction between operations and automation systems, visual perception and motor control of operators are usually modeled with existing equations, and the complex component of operators’ decision-making is the research gap to be addressed in the new model.

**Step 3: Development of A New Mathematical Model Based on Exiting Models to Bridge the Research Gap**

The third step includes the development of the new equations and model component based on the integration and modification of the existing equations depending on the research gap to be bridged. Based on the last step, the inputs and outputs of the model will be identified. The new equations will be built with existing mathematical models or the mathematical formulation of the existing conceptual models and psychological theories from the literature to address the missing components in the targeted problem. To be more specific, the mathematical functions are built based on the theoretical positive/negative and linear/nonlinear relationship between the model inputs and model outputs. Meanwhile, parameters of the existing models that are developed in certain domains can be modified or converted to formulate new models to address the model outputs in the targeted domain. The setting of the parameters can be different in different domains and can be obtained based on the empirical studies in the new domain.

**Step 4: Validation of the Model with Experimental Data**

Lastly, the newly integrated model will be validated by comparing the model prediction and the human data from behavioral experiments. The experiments can be conducted by the researcher themselves or selected from published works. The verification of the model is commonly evaluated via two indexes: R-squared and Root-Mean-Square (RMS) (Wu, 2016). R-squared is used to evaluate how well a model captures the variance of the experimental data. RMS reflects the absolute difference between the model prediction and experimental data. In addition, the statistical analysis method can be used to examine the significance of the differences between models’ predictions and experimental data. If a significant difference was found, the model needs to be further improved.

**CASE STUDY**

Following the abovementioned four steps, this section will present a case study to explain how a mathematical model and equations are built to quantify a specific aspect or component of human cognition and performance in details. A research problem was firstly proposed that we are interested in developing a model to quantify the effect of warning lead time on drivers’ braking performance.

**Step 1: Review of Existing Mathematical Driver Models**

Based on step 1, a comprehensive literature review of mathematical models of driver behavior and performance was conducted. By checking the abovementioned website of QN-MHP, the results showed the existing models has successfully quantified different aspects of driver behavior and performance, including speed control, lateral control and lane change, driver distraction, driver workload, and driver reaction time (Wu, 2016). Besides the mathematical model built with QN-MHP, there are other computational models that addressed drivers’ longitudinal and lateral control (e.g., Salvucci, 2006). Although these models are simulation models rather than mathematical models, the equations within these models provided insights on the development of driver models.

![Figure 2. The working flow of the mathematical model integration and development.](image)

As it shown in the left column of Figure 2, to summarize the results of the literature review, most of the existing driver models focused on the modeling of driver longitudinal and later control performance in normal driving situations (Zhao and Wu, 2013) rather than in emergency conditions. One of the exceptions is the model developed by Zhang, Wu & Wan (2016) that quantified driver response time in speech warning responses. However, the performance of the driver braking
responses was not modeled with the influence of warning technology. There was still a lack of mathematical models that predict braking performance in collision-avoidance situations and quantify the effects of the warning characteristics.

**Step 2: Identification of the Research Gap between Existing Driver Models and the Model of Driver Braking Performance in Warning Responses**

Following the step 2, the research gap was identified. The research gap in this modeling work was associated with the transformation of the speed control model in normal driving conditions to the braking performance model under emergency conditions. Moreover, the effects of warning lead time on the braking performance needed to be quantified. Therefore, the inputs of the model included the warning characteristics (i.e., warning lead time) and the vehicle variables, and the outputs of the model were the brake pedal angle and the deceleration.

**Step 3: Development of A New Mathematical Driver Model of Braking Performance in Warning Responses**

Entering the step 3, the braking performance model with the effect of warning lead time was built and integrated with the existing speed control model in normal driving situations to fill the identified research gap. Zhao and Wu (2013) proposed and validated a mathematical driver speed control model in normal driving conditions by integrating the QN-MHP architecture with the rule-based decision field theory. This model predicted the change of pedal angles in drivers’ acceleration-deceleration processes by modeling drivers’ speed perception based on visual cues and drivers’ decision making on speed choice. The change of brake pedal angle ($\omega$) was defined as a function of the difference between perceived current speed and target speed at the time $t$ as:

$$d\theta = A \times \eta \times (v_{tar} - v_p) \int dt \quad \text{(Zhao and Wu, 2013)}$$

where $v_{tar}$ is the target speed and $v_p$ is the perceived current speed. $A$ is estimated as 0.39 and $\eta$ is estimated as 1 for an averaged driver (Zhao et al., 2013).

The warning effect on motor speed control was reflected by the response time of a warning and the pedal angular velocity. In a warning response task, the warning needs to be processed by a driver to initiate the braking response. Therefore, a time period ($T_r$) was added into the model of the brake pedal angle in order to quantify the delay in drivers’ warning response. Therefore, the model of brake pedal angle at the next time point was modified as:

$$d\theta = A \times \eta \times (v_{tar} - v_p) \int (t - T_r) \quad \text{(2)}$$

where $T_r$ denotes warning response time. In the warning braking response, the target speed is zero ($v_{tar}=0$) since the drivers aim to decelerate to stop before the hazard location. Zhang, Wu & Wan (2016) has developed and validated a model to predict the warning response time ($T_r$). This model quantified warning performance time by modeling the route choice of information processing at humans’ phonological loop with different probabilities ($P_i$ and $P_{II}$).

$$T_r = [T_s + T_g + T_b + T_p + T_{r} + T_{II}] \times P_i + [T_s + T_g + T_b + T_p + T_{r} + T_{II}] \times P_{II}$$

where $T_i (k=5-8, B, C, F, W-Z)$ is the processing time at each server within QN-MHP. $P_i$ and $P_{II}$ are probabilities of a warning stimulus traveling via Route I (the shorter route) and Route II (the longer route), respectively.

Next, the variable of warning lead time ($t_w$) was introduced to model the warning effect on pedal angular velocity. A longer lead time was associated with less emergent situations in which drivers could stop the vehicle with a moderate pressing of braking pedals, whereas a shorter lead time was associated with emergent situations in which drivers could only stop the vehicles with a hard pressing of the braking pedals. Therefore, the model of the brake pedals changes in equation (2) was updated by introducing the effect of warning lead time with a reverse function in equation (4),

$$d\theta = A \times \eta \times \frac{1}{t_w} \times (v_{tar} - v_p) \int (t - T_r)$$

where $t_w$ denotes warning lead time. $v_{tar}$ is the target speed and $v_p$ is the perceived current speed. $T_r$ denotes warning response time in the warning braking response. $A$ is estimated as 0.39 and $\eta$ is estimated as 1 for an averaged driver (Zhao et al., 2013).

The acceleration is estimated to be proportional to the pedal angle in the model developed by Zhao and Wu (2013).

$$da = B \times d\theta$$

where $B$ is the coefficient that represents the ratio of the deviation of the acceleration over the deviation of the pedal input ($B=1.53$, Zhao et al., 2013).

Based on above model, the equation of acceleration ($a$) in warning response was proposed in the same format as the acceleration model in Zhao et al.’s work by predicting the effect of warning lead time and adding the delay of warning response time.

$$da = B \times A \times \eta \times \frac{1}{t_w} \times (v_{tar} - v_p) \int (t - T_r)$$

where $t_w$ denotes warning lead time. $v_{tar}$ is the target speed and $v_p$ is the perceived current speed. $T_r$ denotes warning response time in the warning braking response. $B$ is the coefficient that is estimated as -1.53, $A$ is estimated as 0.39, and $\eta$ is estimated as 1 for an averaged driver (Zhao et al., 2013).

**Step 4: Validation of the Driver Model with Experimental Data**

In step 4, the developed model was validated by comparing the model predictions with experimental data. A driving simulator experiment was designed to study the effects of warning lead time on driver braking performance.

**Participants:** Sixteen participants (11 males, 5 females) with ages ranging from 18 to 29 years participated the study. The inclusion criteria for the experiment included English speaking and a valid US driver’s license.

**Experiment Design:** The current experiment adopted a one-factor experiment design with lead time as an independent variable and the deceleration in braking response as a dependent variable. The lead time was designed with two levels (2.5 s vs. 4.5 s) to represent different levels of collision event urgency. 4.5s represents the optimal warning lead time that leads to the most of the safety benefits (Wan, Wu, &
Zhang, 2016) and 2.5s represents the necessary warning lead time that at least allows drivers to react to warnings before the collision (Yan, Zhang, & Ma, 2015). The order of the lead time was balanced. Eight different scenarios were programmed to represent the common collision events in the real world. The order of collision events was randomized.

Scenario Setting: The speech warning would be presented when a hazard occurred. Each speech warning started with a signal word “Danger” and followed by a description of the collision scenario. Normal road events (e.g., the emergence and departure of a lead vehicle) were designed and randomly assigned between two collision events to eliminate the learning effect. To prevent drivers from associating hazards with the emergence of auditory information, twenty normal messages (e.g., news) were presented to drivers randomly during the test block with same speech rate and loudness level of the warnings.

Apparatus. The driver performance was tested and measured via a STISIM® driving simulator (STISIMDRIVE M100 K, Systems Technology Inc., Hawthorne, CA), including a Logitech Momo® steering wheel with force feedback (Logitech Inc., Fremont, CA), a throttle pedal, and a brake pedal. Driving scenarios were presented on a 27-inch LCD with 1920 × 1200 pixel resolution. A set of speakers was programmed to represent the common collision even

DISCUSSION

The current paper provided a tutorial to develop new models and equations based on the integration of existing mathematical models. The steps in mathematical model integration and development were firstly introduced and explained in general. A case study was presented by following each step to present how the literature of models was reviewed, how the research gap was identified, and how a mathematical modeling was developed and validated. This case study presented the development process of a new model of drivers’ braking performance in warning responses with the integration with the driver speed control model and the warning response time model, and its validation by comparing the model prediction with the behavioral data from an experimental study. This was the first work that provided a detailed explanation of the steps in mathematical modeling development. It could be used as guidance for human factors professionals to learn the mathematical modeling approaches and will benefit the field of human performance modeling.

REFERENCE


