Astr 545 – Astrophysics of Stars and Accretion – Fall 2015

Homework #3

Due Date: Thursday October 8, 2015, 1:59pm

1. Empirical binding energy formula. Using the approximate Weizsäcker mass formula, find the nucleus with the maximum binding energy per nucleon.
(a) As a first approximation, you may assume that the numbers of protons and neutrons in the nucleus are equal.
(b) Use the relation between the numbers of protons and neutrons along the beta-stability line, as given in the lectures, to repeat the above exercise. Here, you will need to solve the resulting algebraic equation numerically.
(c) Plot the binding energy as a function of atomic number for the most stable isotope of each chemical element as predicted by the Weizcäcker mass formula. Overplot on the same graph the measured binding energies of the elements obtained by the experiments reported in "The Ame2003 atomic mass evaluation (II)" by G.Audi, A.H.Wapstra, & C.Thibault Nuclear Physics A729 p. 337-676 (you can find the data in an electronic form at
`http://www.nndc.bnl.gov/masses/mass.mas03`)
(you may use only a representative number of elements that cover the whole range of atomic numbers). Discuss your results.

2. Scaling Relations for Stars in the Main Sequence

In this problem, you will try to guess the overall scaling of various physical quantities of stars in the Main Sequence, and then compare them to more detailed analytic models, as well as to numerical models.

Follow simple dimensional arguments to derive:

1. A relation between the mass of a star, its radius, and its central density.
2. A relation between the radius of a star, its mass, its central density, and its central temperature, using the equation of hydrostatic equilibrium.
3. A relation between the luminosity of a hydrogen burning star, its central density, its central density, the Gamow energy for hydrogen burning, and other necessary micro-physical quantities.
4. A relation between the luminosity of the star, its radius, its central density, and its central temperature, assuming a fully radiative interior. How would this relation be different had the star been fully convective?

Use the above relations to derive an appropriate scaling for the dependence of stellar radius on mass and of luminosity on mass for main sequence stars.

Compare now this result of dimensional analysis to the polytropic models presented in the paper by F. Adams, arXiv:0807.3697. Did you find anything missing, other than dimensionless quantities of order unity? What is the limitation of the dimensional analysis, even compared to more detailed analytic calculations?

Use the luminosity-mass relation you obtained to derive a scaling for the main-sequence lifetime of a star as a function of its mass. Compare it to the numerical result of Schaler et al. 1992, A&A Supp., 96, 269 (use the table for Z=0.02). What do you observe? What is the limitation of the analytic calculation compared to the numerical one?