Astr 545 – Astrophysics of Stars and Accretion – Fall 2015

Homework #4

Due Date: Thursday October 29, 2015, 1:59pm

1. The 3α reaction. The energy generation rate in the 3α reaction is given by

$$\epsilon_{3\alpha} \simeq 3.9 \times 10^{11} \frac{\rho^2 Y^3}{T_8^3} f \exp\left(-\frac{42.94}{T_8}\right) \text{ erg g}^{-1}\text{s}^{-1} , \tag{1}$$

where ρ is the plasma density, $T_8 = T/10^8$ K is the temperature, Y is the helium mass fraction, and f is the electron screening factor.

(a) Plot the quantity $\epsilon_{3\alpha}/(\rho^2 Y^3 f)$ as a function of temperature.

(b) Show that for small temperature variations around some temperature T_0, the energy generation rate can be approximated by

$$\epsilon_{3\alpha} = \epsilon_{3\alpha 0}(T_0)\left(\frac{T}{T_0}\right)^n , \tag{2}$$

where

$$n = \frac{42.94}{T_8} - 3 . \tag{3}$$

(c) Show that for $T_8 \simeq 1$, the energy generation rate is approximately

$$\epsilon_{3\alpha} \simeq 4.4 \times 10^{-8} \rho^2 Y^3 f T_8^{40} \text{ erg g}^{-1}\text{s}^{-1} , \tag{4}$$

and plot this approximate expression on top of the curve for the complete expression that you plotted in part (a). This rapid dependence of the 3α reaction on temperature (together with the fact that it often takes place in degenerate conditions) is responsible for many of the energetic flashes in astrophysical objects.

2. The Isothermal Wind. Consider the case of an isothermal wind (i.e., $\gamma = 1$). This is one of the special cases that we did not consider in class and, therefore, some of the equations we used are not valid but need to be rederived.

(a) Start by deriving the basic equation

$$\frac{1}{2}\left(\frac{u^2 - c^2}{u^2}\right)\frac{du^2}{dr} = \frac{2c^2}{r} - \frac{GM}{r^2} , \tag{5}$$

where u and c are the radial velocity and isothermal sound speed of the material, respectively, and M is the mass of the central object.

(b) Defining the isothermal mach number as $\mathcal{M} \equiv u/c$, show that the solution of this equation is

$$\mathcal{M}^2 - \ln \mathcal{M}^2 = 4 \ln\left(\frac{r}{r_c}\right) + 4\left(\frac{r}{r_c}\right)^{-1} + \text{constant} , \tag{6}$$

where r_c is the location of the critical point.

(c) Plot the solution to the above equation for different values of the constant of intergration and argue which of those solutions can represent physically plausible profiles for isothermal winds.