Astr 545 – Astrophysics of Stars and Accretion – Fall 2011

What you need to know coming out of this class

The Equations of Fluid Mechanics

1. Under what conditions can you use the equations of fluid mechanics to model a physical system? When do you have to resort to solving the Boltzmann equation?
2. Describe at least one astrophysical setting, modeling of which requires solving the Boltzmann equation instead of the fluid equations.
3. What is the physical meaning of the pressure gradient and the viscosity terms in the Euler equation, when the fluid is collisionless?
4. What is the closure problem in deriving the equations of fluid mechanics? How is it addressed in the absence of turbulent motions?
5. Use dimensional arguments to derive an approximate expression for the coefficient of shear viscosity in a plasma in terms of its density, the sound speed, and a characteristic length scale. What is this characteristic length scale?

The Equation of State

6. Under what conditions can you use the Saha equation to calculate ionization balance in a plasma?
7. A straightforward evaluation of the Saha equation for the conditions in the center of the Sun ($T = 1.5 \times 10^7$ K and $\rho = 100$ g cm^{-3}) leads to the conclusion that hydrogen is only 75% ionized there. Can this be correct?
8. Describe at least two physical phenomena that cause the equation of state of a non-degenerate, non-relativistic plasma to deviate from that of an ideal gas.
9. What is the condition that needs to be satisfied for relativistic effects to be negligible in calculations of the equation of state of a plasma?
10. Provide a reasonable condition that allows you to check whether a fully ionized hydrogen plasma is degenerate.

Polytropes

11. What is the difference between a polytropic and an adiabatic equation of state. What determines the polytropic index in the first case and the adiabatic index in the second?
12. Describe two astrophysical settings in which a polytropic or an adiabatic equation of state is exact.
13. Use the fact that the radius of a polytropic object with index $n \geq 5$ is infinite to show that the mass in an isothermal sphere in hydrostatic equilibrium is also infinite. The polytropic index is the exponent in the relation $P = K\rho^{1+1/n}$.

The Radiation Field

14. Define the specific intensity of the radiation field in terms of the photon occupation number in phase space. What are the units of the specific intensity?

15. In an astrophysical context, what is the difference between absorption/emission and scattering of photons by matter? In practice, what is the fundamental difference between the terms that describe absorption/emission and scattering in the radiative transfer equation?

16. Consider a radiation hydrodynamics problem with *(a)* spherical symmetry, *(b)* axisymmetry, and *(c)* no symmetry at all. How many phase-space dimensions do you need to consider in order to describe completely the radiation field in each case?

17. Define the zeroth and first moments of the specific intensity of a radiation field and describe their physical meanings.

18. Write a general expression for the radiation force in terms of the specific intensity of the radiation field and the coefficients for absorption and scattering. Under what conditions does the radiation force behave as a gradient of a "radiation pressure"?

19. Show that, in a plane-parallel atmosphere, the specific intensity I in the diffusion regime can be written as

    $$I = J + 3\mu H \ ,$$

    where J and H are its zeroth and first moments and $\mu \equiv \cos\theta$ is the cosine of the angle of propagation of photons with respect to the symmetry axis of the problem.

20. Calculate the variable Eddington factor $f \equiv K/J$, where J and K are the zeroth and second moments of the specific intensity of the radiation field in a spherically symmetric system and show that $f = 1$ in the free-streaming regime and $f = 1/3$ in the diffusion regime. What do your results imply for the radiation force exerted on matter in the two regimes?

21. The source function in a plasma is defined as the ratio of the emissivity η_ν to the absorption coefficient χ_ν

    $$S_\nu \equiv \frac{\eta_\nu}{\chi_\nu} \ .$$

    Under what conditions is the source function equal to the blackbody function, i.e., under what conditions is Kirchhoff's law valid?

22. Draw the specific intensity for blackbodies with different temperatures as a function of photon frequency. Show explicitly the asymptotic behavior of the curves at the two asymptotic regimes (i.e., very low and very high frequencies) and mark the frequency of the peak of the specific intensity in each case.

23. What are the Planck and Rosseland mean opacities? Why are they useful in generating models of stellar atmospheres?

24. Define the following interactions between photons and matter and, in each case, discuss qualitatively the dependence of the cross section of interaction on the density of matter and on photon frequency: *(a)* electron scattering, *(b)* bremsstrahlung, *(c)* bound-free transitions, *(d)* bound-bound transitions.

Models of Stellar Atmospheres

25. Which conditions need to be met for a stellar atmosphere to be in *(a)* radiative equilibrium or *(b)* local thermodynamic equilibrium? What do these conditions imply for *(i)* the occupation of different atomic states, *(ii)* the distribution of "random" velocities in the plasma, and *(iii)* the energy distribution of photons, in each case?

26. What determines the temperature profile in an atmosphere in radiative equilibrium? (Fick's law)

27. Derive the formal solution to the radiative transfer equation. Under what conditions is this solution valid?

28. Use the formal solution to the radiative transfer equation to find the explicit conditions under which limb darkening will be present in an atmosphere in radiative equilibrium.

29. What is the Λ-iteration scheme for calculating model atmospheres in radiative equilibrium? Why should you never use it? Describe at least one technique that allows construction of model atmospheres in radiative equilibrium.

30. Discuss briefly each of the following line-broadening mechanisms: *(a)* natural broadening, *(b)* thermal Doppler broadening, *(c)* rotational broadening, *(d)* pressure broadening. In each case, what is the profile of the broadened line and what determines its width?

31. What is the Voigt line profile? What physical mechanisms are responsible for it?

Nuclear Astrophysics

32. Use simple dimensional arguments to show that the cross section of interaction between nuclei (for non-resonant reactions) should scale with energy E as

$$\sigma(E) = \frac{S(E)}{E} \exp\left(-bE^{-1/2}\right) ,$$

where $S(E)$ is a slowly varying function of energy and b is a constant.

33. Use the above expression to show that the reaction rate between nuclei with thermal energy distributions is sharply peaked at an energy

$$E_0 = \left(\frac{bk_\mathrm{B}T}{2}\right)^{2/3} ,$$

where k_B is Boltzmann's constant and T is the temperature of the thermal distribution. Explain in simple physical terms the reason responsible for this sharp peak.

34. Use the expression for E_0 to show that, for each nuclear reaction, there is a characteristic temperature at which the reaction is most efficient. What is this temperature for hydrogen burning?

35. What is the valley of β-stability in the Segre chart of isotopes? Why is the β-stability valley skewed towards more neutron-heavy nuclei at large atomic numbers?

36. What are the basic characteristics of the *pp*-cycle and the CNO-cycle in main sequence stars? How does the stellar metalicity and the mass of the star affect the reaction rates in each cycle (qualitatively)? For which stars does the *pp*-cycle dominate?

37. Why does He-burning not proceed via normal proton or α-particle capture but has to wait until the temperature and density in the center of the star becomes high enough for the 3α process to occur?

38. What is the heaviest element that can be produced inside stars by reactions that involve charged particles? What are the two major processes with which heavier nuclei are produced?

39. What is the $s-$process nucleosynthesis? What is the $r-$process nucleosyntesis? What is the heaviest element that can be produced with either process? What are the astrophysical settings where these processes are believe to take place?

Stellar Models

40. Under what conditions is a stratified plasma unstable to convection? What are the different criteria when there is (Ledoux) and when there isn't (Schwarzschild) a gradient in the elemental composition?

41. What is semi-convection (or doubly-diffusive convection)?

42. Describe briefly the mixing-length model of convection. What is the benefit of using this model? What are the drawbacks?

43. Explain why it is often reasonable to neglect stellar rotation in constructing models of stellar winds.

44. What is the fundamental difference between radiation-driven and thermally-driven winds? What is the behavior of the asymptotic velocity (i.e., the velocity at large distances) in the two cases and why?

45. Draw the Mach number as a function of radius for the various solutions to the spherically symmetric, thermally driven wind problem. Identify the sonic point and discuss its significance. Point out the solution that corresponds to a realistic stellar wind.

46. What is the characteristic shape of an atomic line from star that has a strong wind (p Cygni profile). What determines how strong the asymmetry of the line is?

47. How would you describe the rotational properties of the Sun, both in terms of latitudinal variations as well as at different depths?

48. What do we know about the rotation rates of main sequence stars of different spectral types? What do we know about the evolution of rotation rates of stars with age?

49. How and when is the rotation of a star affected by the presence of a binary companion?

50. Describe at least three effects of rapid rotation on the structure of a star.

51. According to von Zeipel's theorem, the local flux of radiation through an equipotential surface in a rotating star is proportional to the local gravitational acceleration. Use this theorem to show that *(a)* the effective temperature on the surface of a rotating star is higher on the pole than on the equator and *(b)* the star cannot be at the same time in hydrodynamic and radiative equilibrium (von Zeipel's paradox). How do rotating stars in the Universe avoid von Zeipel's paradox?

52. Give simple dimensional arguments to show that the typical mass of a star in our Universe can be expressed in terms of fundamental constants as

$$M \simeq \left(\frac{\hbar c}{G}\right)^{3/2} m_p^{-2} \simeq 1.85 M_\odot ,$$

where $\hbar$ is Planck's constant, c is the speed of light, G is the gravitational constant, and m_p is the mass of the proton.

53. What determines the minimum mass of a star for hydrogen burning to occur in its core? (hint: the answer is not simply that the temperature is too low for H-burning to ignite. what stops the star from collapsing further then and increase its central temperature?) What is this minimum mass (approximately)?

54. What determines the maximum mass of a stable star in the main sequence? What is that maximum mass (approximately)? How does the composition of the star affect this result?

Stellar Evolution

55. Draw an HR diagram and place on it the location of the main sequence. Describe briefly the dominant reaction rate as well as whether the core or the envelope of a star is convective for stars with *(a)* $0.08M_\odot < M < 0.25M_\odot$, *(b)* $0.25M_\odot < M < 1.2M_\odot$, *(c)* $1.2M_\odot < M$.

56. Describe qualitatively why hydrogen burning in the center of a star in the main sequence is a stable process. (hint: what happens when you introduce a small temperature increase in the core?)

57. When does a star leave the main sequence? (hint: the answer is not simply "when it exhausts the hydrogen in its core"; think of the Sönberg-Chandrasekhar instability).

58. How does the main-sequence lifetime of a star depend on its mass and on its metalicity?

59. Draw on an HR diagram the various stages of evolution of a $1M_\odot$ star. Mark each stage on the graph and describe briefly what is the physical phenomenon that drives each stage.

60. What is the helium flash in the evolution of a low-mass star? Why is helium burning unstable there? Why do stars with masses larger than a couple of solar masses avoid the helium flash?

61. Draw on an HR diagram the various stages of evolution of a low-mass star, of an intermediate mass star, and of a massive star. Describe briefly the differences in the evolution and the reasons behind them.

62. What are AGB stars and what are Mira variables?

63. What is the difference between Type Ia, Type Ib, Type Ic, and Type II supernovae, both in their observational appearance and in the physical mechanisms responsible for them?

64. What is the Phillip's relation for Type Ia supernovae and how does it help measurements of the accelerated expansion of the Universe?

65. Describe qualitatively the time evolution of a Type II supernova explosion, from the initial collapse of the pre-supernova star to the breakout of the shock. Describe at last one physical mechanism that has been suggested as responsible for the shock breakout.

End Stages of Stellar Evolution

66. Use the relationship between mass M and central density ρ_c for a polytropic star

$$M \sim \rho_c^{(3-n)/(2-n)} ,$$

where n is the polytropic index, to draw qualitatively the dependence of the mass of a white dwarf on its central density. Use the fact that the polytropic index for non-relativistic degenerate matter is $n = 3/2$ and for relativistic degenerate matter is $n = 3$. Use the plot to define the Chandrasekhar mass limit. What does a realistic curve look like? What determines the maximum white dwarf mass in the realistic case?

67. Radio pulsars are pulsing sources with periods as small as a millisecond. Use simple arguments to show that the densities in the interiors of pulsars have to reach or exceed the nuclear saturation density ($\simeq 3 \times 10^{14}$ g cm-3 and that the radius of a pulsar cannot exceed $\simeq 15 - 20$ km.

68. Radio pulsar magnetic fields are often estimated using the expression for magnetic dipole radiation. Under what conditions is this expression valid? Are these conditions satisfied in the magnetospheres of pulsars?

69. What provides the support against gravitational collapse in a white dwarf and in a neutron star?

70. What determines the maximum mass of a neutron star? How large is it for realistic equations of state?

71. What is the physical difference between the gravitational and baryonic mass of a neutron star? How large is it?

72. Describe at least two observational manifestations of neutron stars in the Universe. How do we know that these objects are neutron stars?

73. Describe at least two observational manifestations of black holes in the Universe. How do we know that these objects are black holes?

Accretion

74. Describe the physical setup of the problem that is often referred to as Bondi-Hoyle accretion. Use dimensional arguments to express the accretion rate in this setup as a function of the mass of the central object, the density and sound speed in the ambient medium, and the velocity of the object through the medium.

75. Use dimensional arguments that express the time it takes for a viscous ring orbiting a central object to be accreted as a function of the distance of the ring from the object and the coefficient of kinematic viscosity.

76. Provide at least one specific argument from observations which proves that molecular viscosity cannot be the dominant source of angular momentum transport in an accretion disk.

77. In a steady-state, geometrically thin, optically thick accretion disk, what determines the rate of mass accretion? what determines the rate with which the angular momentum of the central object increases (these are trick questions!)

78. In a steady-state, geometrically thin, optically thick accretion disk, the flux of radiation emerging from a ring at distance $R >> R_{\rm in}$ is

$$F = \frac{3GM\dot{M}}{4\pi R^3} .$$

How does this flux compare to the potential energy released (per unit surface area) as matter crosses the ring? Where does the extra energy come from?

79. Describe the significance of the innermost stable circular orbit around a black hole in setting, under the standard assumptions, the efficiency of emission from the accretion disk.

80. Calculate the luminosity from an accretion disk at which radiation forces (along the vertical radiation) balance gravity and show that it scales as the spherically-symmetric Eddington luminosity.

81. How does the maximum possible temperature in an accretion disk scale with the mass of the central object? What do you expect this temperature to be for accretion around a $10M_\odot$ and a $10^9M_\odot$ black hole?

82. Given the answer to the previous question, why can we observe AGN in the X-rays?