On Measures of Cooperation in Distributed Systems under Uncertainties

Pavlo A. Krokhmal*[1] David E. Jeffcoat[2]

[1]Department of Mechanical and Industrial Engineering, University of Iowa, Iowa City, IA, 52242
[2]Air Force Research Laboratory/Munitions Directorate, Eglin AFB, FL 32542

June 2010

Abstract

We study the evolution of distributed multi-agent systems under uncertainty where the autonomous agents may cooperate with each other, and/or with supervisor/operator, in order to achieve the system's objective. The cooperation is facilitated by means of information sharing among the autonomous agents and/or supervisor/operator, which has the purpose of improving the effectiveness of the autonomous agents. The evolution of cooperative systems is modeled using discrete-state, continuous-time Markov processes. To measure and quantify the degree of cooperation within such systems, we introduce the concept of coefficient of cooperation, which is obtained by minimizing the Kullback-Leibler or 1-norm distances between nonstationary probability distributions. The presented techniques are illustrated on several different types of multi-agent search systems.

Keywords: Cooperation; multi-agent systems; Markov processes; coefficient of cooperation; Kullback-Leibler divergence.

1 INTRODUCTION

The increasing complexity and sophistication of computing, sensing, and communication technologies paves the way for proliferation of unmanned autonomous systems and platforms, which will replace and/or assist humans in hazardous or resource consuming missions. A key characteristic of such systems is the ability of their various interconnected constituents to cooperate by sharing information and performing joint planning and execution on multiple time and space scales in highly uncertain, dynamic, and volatile conditions. In view of this, the objective of this paper is to develop mathematical techniques for modeling and quantification of the effects of cooperation in distributed systems operating in uncertain environments.

During last decades, an extensive literature has accumulated on the subject of cooperation, particularly in the fields of operations research and game theory, control theory, etc. (see, among others, Butenko et al. (2003); Grundel et al. (2004); Hirsch et al. (2007); Peleg and Sudhölter (2007); Hirsch et al. (2009), and references therein). Most of the existing studies have focused on *how to cooperate*, i.e., on development of decision-making algorithms, policies, and strategies that would facilitate achievement of a "global" goal by a group of autonomous agents that are also pursuing some "local" goals.

*Corresponding author (krokhmal@engineering.uiowa.edu)

The primary objective of the present endeavor is different in that we aim to quantify the effects and benefits of cooperation in distributed systems that operate in uncertain environments. In other words, the focus of our work is not so much on the *mechanisms of cooperation,* but rather on the *impact of cooperation on the overall performance of the distributed system.*

In this work, we consider a class of distributed cooperative systems frequently encountered in military applications, where each member of the system is pursuing its own *local* objective, and the system's *global* objective is considered accomplished when all members of the system have achieved their respective individual (local) goals. In such a setup, cooperation among system's members is crucial for accomplishing their individual objectives and, consequently, the global system objective. Since, in practice, the process of accomplishing a local objective by each of the system's members is not deterministic, and is influenced by a number of stochastic factors, it is considered that cooperation within the system may increase the chances of accomplishing the local goals by the system members. Next, we make an assumption that is key to the proposed approach: a quantifiable increase of chances for a local success by members of the system may occur each time when one of them accomplishes its own local objective (for example, upon finishing its own task, an agent may directly help others in doing so, or may share valuable information/experience with other system members, etc.)

The proposed method for modeling of such cooperative systems under uncertainties employs the techniques of Markov processes. Within this approach, the actual mechanisms of cooperation among the members of the distributed system are not important; instead, we are interested in the degree by which cooperation can improve the chances of accomplishment of a local goal by each individual member, and the corresponding changes in the overall performance of the distributed system as a whole.

As a representative example of a distributed cooperative system, we consider a cooperative search-and-attack mission that involves several cooperative subsystems, such as wide area search munitions, reconnaissance assets, etc. In the context of this task, we will consider information sharing among the members of the autonomous formation as the primary method of communication and cooperation, which has the purpose of increasing the overall effectiveness of the cooperative enterprise. To emphasize that the process of information sharing is focused on improving the situational awareness and operational functionality of the cooperative agents, we call it cueing. However, it must be underlined that the models and methods considered here are quite general, and are applicable to other joint activities where dynamic sharing of information may lead to substantial improvement of joint, coordinated, or cooperative activity.

The present paper is a continuation of the previous research efforts of the authors (Jeffcoat et al., 2006, 2007), where a Markovian framework was applied to modeling of two types of cooperative search systems, with the main emphasis being placed on their asymptotical properties. In particular, it has been shown that, no matter how much cooperation can improve the operational capabilities of the system's individual members, the effects of cooperation on the overall system's performance are inherently bounded, with the bound being controlled by the size of the distributed system and the initial (intrinsic) operational capabilities of its members.

Although the Markovian framework can be considered as somewhat restrictive for modeling of realistic interactions in distributed systems, it does allow for obtaining valuable quantitative and qualitative insights into the characteristic properties of the described cooperative systems, which may not be available by using other modeling methods, such as simulation, etc.

The objective of the present work is to extend the techniques introduced in Jeffcoat et al. (2006, 2007) and develop an approach to measurement and quantification of the degree of cooperation within distributed systems, i.e., to answer the questions "how cooperative is the given system?" and "is the given system more cooperative than the other?"

The paper is organized as follows. In the next section we introduce several types of cooperative systems, where the autonomous searchers may cooperate with each other and/or supervisor/operator by sharing information. The systems are modeled as discrete-state continuous-time Markov processes, and the corresponding sets of Chapman-Kolmogorov differential equations are derived. Section 3 presents numerical results on the models considered in Section 2. Finally, Section 4 discusses an approach to measuring the degree of cooperation within a distributed system, and illustrates it on the examples of the search systems discussed in the paper.

2 MODELING OF COOPERATIVE SYSTEMS: A SEARCH MISSION EXAMPLE

The particular type of distributed cooperative system that we focus on in this work is the search system, where N autonomous agents, or searchers, are given the objective of discovering (detecting) a certain kind of objects of interest, or targets.

In the simplest case, each of N searchers may assume only two states: 1 (*"Search"*) or 2 (*"Detect"*), see Fig. 1. Once a searcher reaches the *"Detect"* state (i.e., detects a target), it never returns to the *"Search"* state (i.e., does not resume the search). Such a setup is common to search-and-rescue missions, where, upon detecting a target searchers would try to perform a rescue operation instead of continuing the search; it is also applicable to search-and-attack missions involving multiple expendable wide-area search munitions. The effectiveness of individual searchers is characterized by the detection rate ϑ_{12}, i.e. the probability of detecting a target within time interval Δt:

$$\mathsf{P}\big\{\text{searcher detects a target during time } \Delta t\big\} = \vartheta_{12}\Delta t + o(\Delta t). \tag{1}$$

Then, the conditional probability of target detection at time t, given that the searcher is in the *"Search"* state, has exponential distribution with mean ϑ_{12}^{-1}:

$$f(t;\vartheta_{12}) = 1 - e^{-\vartheta_{12}t}, \quad t \geq 0. \tag{2}$$

The exponential distribution in (2) is prerequisite for the Markovian models of cooperative search systems that are presented next. Recall that in the systems with Markov properties, the exponential distribution describes the time elapsed between "rare" events, and is closely connected to the Poisson distribution of the number of "rare" events per unit of time (see, e.g., Papoulis and Pillai, 2002). Given that in the context of a search mission the detection events can be considered as "rare" (in the sense that the probability of making a detection at any time instant is small, see (1)), the assumption of exponential distribution does not appear particularly restrictive.

The cooperation within the search system is facilitated by information sharing, or *cueing*, among the searchers, and has the purpose of increasing their detection capabilities. The informational content of the cueing signals is not important in the presented framework; instead, we are interested in the degree by which cueing impacts the search capabilities of individual agents in a cooperative system.

Below we discuss several cooperative search systems with various forms of cooperation via information sharing, which serves as a mechanism to achieve a better system performance.

2.1 An unsupervised cooperative search model

We start with the simplest search system, where the autonomous agents are acting cooperatively and without supervision. It is assumed that upon detecting a target and transitioning to the state *"Detect"*, the searcher

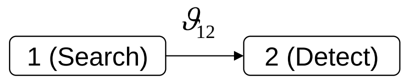

Figure 1: States of an autonomous searcher

instantaneously cues (shares information with) all other searchers that are still in the *"Search"* state, thereby potentially increasing their detection capabilities (for instance, the searcher that made detection may communicate information that narrows down search area, or locations of high-priority targets, etc). Again, the details of the cooperation mechanism are not essential within the presented framework; it is only assumed that, as a result of cooperation, the detection rates of searchers assume values

$$\vartheta_{12} = \theta^* + c_k, \tag{3}$$

where θ^* is the *intrinsic* detection rate of a searcher, and $c_k \geq 0$ is a gain in the detection rate due to cooperation that is attained when k searchers made a detection. In other words, the searchers start the mission having the same (intrinsic) detection rate θ^*; as soon as one of them makes a detection, it shares some information (search direction, target locations, etc.) with other team members, thereby increasing their detection rates to $\theta^* + c_1$; after another searcher makes a detection, the detection rates of the remaining searchers improve to $\theta^* + c_2$, and so on. In short, (3) is the rate common to $N - k$ searchers that are still in the *"Search"* state (obviously, one has $c_0 = 0$).

By introducing the traditional assumptions of independence of target detections on non-overlapping time intervals etc., one can model the outlined cooperative system as a continuous-time discrete-state Markov process (see, among others, Brémaud, 1999; Papoulis and Pillai, 2002). The states $\mathcal{S}_{ik}$ of the cooperative system are determined by the numbers i and k of agents that are in the state *"Search"* and *"Detect"*, respectively. By defining the state probabilities $\pi_{ik}(t)$ as

$$\pi_{ik}(t) = \mathsf{P}\big\{\text{system is in one of the states } \mathcal{S}_{ik} \text{ at time } t\big\},$$

one can write the system of backward Chapman-Kolmogorov ODEs governing these probabilities:

$$\frac{\mathrm{d}}{\mathrm{d}t}\pi_{ik}(t) = -i(\theta^* + c_k)\pi_{ik}(t) + (i+1)(\theta^* + c_{k-1})\pi_{i+1,k-1}(t), \quad i + k = N. \tag{4}$$

Equations (4) can be stated more conveniently by denoting $\pi_{ik}(t) = p_k(t)$ for $k = 0, \dots, N$, so that the above system takes the form

$$\frac{\mathrm{d}}{\mathrm{d}t}p_k(t) = -(N-k)(\theta^* + c_k)p_k(t) + (N-k+1)(\theta^* + c_{k-1})p_{k-1}(t), \quad k = 0, \dots, N. \tag{5}$$

Equations (4) and (5) have a simple interpretation: the probability of the system occupying the state $\mathcal{S}_{ik}$ decreases at the rate $(N-k)(\theta^* + c_k)p_k(t)$ as each of $i = N - k$ agents that are in the state *"Search"* may detect a target and thus the system will transition into the state $\mathcal{S}_{i-1,k+1}$. On the other hand, probability $p_k(t)$ of the cooperative system occupying state $\mathcal{S}_{ik}$ increases at the rate $(N-k+1)(\theta^* + c_{k-1})p_{k-1}(t)$ due to the possibility of a transition from the state $\mathcal{S}_{i+1,k-1}$ where $i+1$ agents are in the state *"Search"*.

The initial conditions for equations (4)–(5) reflect the fact that at $t = 0$ the system is in the state $\mathcal{S}_{N0}$ with probability 1:

$$\pi_{ik}(0) = p_k(0) = \delta_{k0}, \quad k = 0, \dots, N, \tag{6}$$

where δ_{ij} is the Kronecker's delta. Using (5) and (6) it is straightforward to verify that the probabilities $p_k(t)$ satisfy the identity

$$\sum_{k=0}^{N} p_k(t) = 1, \quad t \geq 0. \tag{7}$$

Below we consider several models of supervised cooperative search systems, where the supervisor may be either a human operator or another higher-level autonomous agent. Depending on the functions of the supervisor, the members of the cooperative system will have additional states that correspond to the control actions of the supervisor. In the next subsections we consider several models where the members of supervised cooperative system have three states.

2.2 A coordinated search model

In this subsection we consider a distributed multi-agent system where the search is coordinated by a supervisor (e.g., human operator). It is assumed that the supervisor conducts search for targets independently of the autonomous agents, and, upon discovery of appropriate objects of interest, directs one of the searchers to this target. In effect, each searcher is modeled using a Markov process with three states: *"Search"*, *"Search directed"*, and *"Detect"* (see Fig. 2). The transition from the state *"Search"* to *"Search directed"* depends on the search rate λ of the supervisor, which is assumed to cue only one searcher at a time, and that the cues are distributed uniformly[1] to the uncued searches. This implies that the transition rate from *"Search"* to *"Search directed"* equals to

$$\vartheta_{12} = \bar{\delta}_{i0} \frac{\lambda}{i},$$

where i is the number of searchers in the state *"Search"*, and $\bar{\delta}_{ij}$ is the negation of the Kronecker delta:

$$\bar{\delta}_{ij} = 1 - \delta_{ij} = \begin{cases} 0, & \text{if } i = j, \\ 1, & \text{if } i \neq j. \end{cases}$$

After being directed by the supervisor, the searcher transitions to the state *"Detect"* at a rate

$$\vartheta_{23} = \theta^* + c_{\mathrm{d}},$$

where, as before, θ^* is the intrinsic detection rate of a searcher, and $c_{\mathrm{d}} > 0$ is the gain in the detection rate due to the information received from the coordinator/supervisor. The autonomous searchers also have the ability to search independently, thus they can make a direct transition from the state *"Search"* to the *"Detect"* state at a rate $\vartheta_{13} = \theta^*$. Since in this setting the searchers do not cooperate directly with each other, we call this model a *coordinated* search system.

Similarly to the exposition of Section 2.1, let $\mathcal{S}_{ijk}$ be the state of the system in which there are i searchers in the state *"Search"*, j searchers in the state *"Search directed"*, and k searchers in the state *"Detect"*. It is easy to see that there are $\binom{N+3-1}{N} = \frac{(N+2)(N+1)}{2}$ states $\mathcal{S}_{ijk}$ such that $i + j + k = N$. By defining the state probabilities as

$$\pi_{ijk}(t) = \mathsf{P}\{\text{the system is in state } \mathcal{S}_{ijk} \text{ at time } t\}, \tag{8}$$

[1]This assumption implies that the supervisor is aware of the current state of all the searchers and can transmit information to a single searcher. Even if a transmission is broadcast on a common frequency, we assume that the transmitted data can be "tagged" for use only by an individual searcher. The uniform distribution of cues follows from the symmetry considerations, since all searchers in the *"Search"* state are equally likely to be picked as the next recipient of a cue from the supervisor.

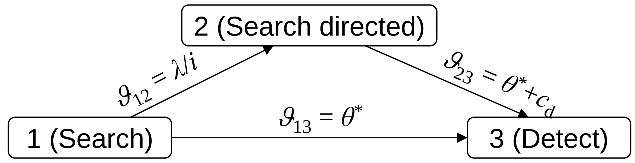

Figure 2: State diagram for an autonomous searcher in the coordinated search system

one can describe the corresponding Markov model with a finite number of states via the following system of Chapman-Kolmogorov equations:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\pi_{ijk}(t) = &-\bar{\delta}_{kN}\Big[i\theta^{*} + j(\theta^{*} + c_{\mathrm{d}}) + \bar{\delta}_{i0}\lambda\Big]\pi_{ijk}(t) + \bar{\delta}_{iN}\bar{\delta}_{j0}\lambda\,\pi_{i+1,\,j-1,\,k}(t)\\ &+ \bar{\delta}_{iN}\bar{\delta}_{k0}(i+1)\theta^{*}\pi_{i+1,\,j,\,k-1}(t) + \bar{\delta}_{jN}\bar{\delta}_{k0}(j+1)(\theta^{*} + c_{\mathrm{d}})\,\pi_{i,\,j+1,\,k-1}(t),\\ &\qquad\qquad i+j+k = N,\end{aligned} \tag{9}$$

where the factors $\bar{\delta}_{ij}$ have the obvious function of handling the extreme cases of i, j, or k being equal to 0 or N. Let us present the interpretation of equations (9). In the most general case the state $\mathcal{S}_{ijk}$ can be obtained

- from the state $\mathcal{S}_{i+1,j-1,k}$ due to a transition *"Search"*$\rightarrow$ *"Search directed"*, i.e. when one of the $i+1$ searchers receives a directive from the operator. Since each of the $i+1$ searchers in the state *"Search"* is being cued by the operator at a rate $\frac{\bar{\delta}_{i+1,0}}{i+1}\lambda$, the transitions *"Search"* $\rightarrow$ *"Search directed"* increase the probability $\pi_{ijk}(t)$ at the rate $\lambda\pi_{i+1,j-1,k}(t)$. This amounts to the second term in equation (9).

- from the state $\mathcal{S}_{i+1,j,k-1}$ due to a transition *"Search"* $\rightarrow$ *"Detect"*, i.e., when one of the $i+1$ searchers detects a target without a directive from the operator. The search rate of each of $i+1$ "undirected" agents is θ^{*}, thus due to transitions *"Search"* $\rightarrow$ *"Detect"* the probability $\pi_{ijk}(t)$ increases at the rate $(i+1)\theta^{*}\pi_{i+1,j,k-1}(t)$, which amounts to the third term in (9).

- from the state $\mathcal{S}_{i,j+1,k-1}$ due to a transition *"Search directed"* $\rightarrow$ *"Detect"*, when one of the $j+1$ directed searchers detects a target. The search rate of a directed agent is $\theta^{*} + c_{\mathrm{d}}$, whereby transitions *"Search directed"* $\rightarrow$ *"Detect"* increase the probability $\pi_{ijk}(t)$ at the rate $(j+1)(\theta^{*} + c_{\mathrm{d}})\pi_{i,j+1,k-1}(t)$, which amounts to the fourth term in (9).

- finally, the first term in the right-hand side of (9) accounts for the possibility of transition from the given state $\mathcal{S}_{ijk}$ to states $\mathcal{S}_{i-1,j,k+1}$, $\mathcal{S}_{i,j-1,k+1}$, and $\mathcal{S}_{i-1,j+1,k}$, correspondingly.

At $t=0$ the system is in the state $\mathcal{S}_{N00}$, thus the initial conditions for equations (9) are

$$\pi_{ijk} = \delta_{iN}, \quad i+j+K = N, \tag{10}$$

and, similarly to (7), solutions of (9) satisfy the identity

$$\sum_{\substack{j+j+k=N\\ i,j,k\geq 0}} \pi_{ijk}(t) = 1, \quad t \geq 0. \tag{11}$$

2.3 Coordinated cooperative search model

Here we consider a generalization of the search systems presented in Sections 2.1 and 2.2, with the autonomous searchers being capable of receiving cues from an operator as well as sending cues to each other upon target detection. Similarly to the coordinated search model of Section 2.2, each autonomous searcher has three states, *"Search"*, *"Search directed"*, and *"Detect"* (see Fig. 3), and i, j, and k denote the number of autonomous searchers in these states, respectively.

In contrast to the model considered in Section 2.2, the autonomous searchers can cooperate with each other via cueing as described in Section 2.1, which generally leads to improvement of their detection rates in accordance to (3). Consequently, the detection rate of each of i searchers in the state *"Search"* is given by (3) and equals to $\theta^* + c_k$, where k is the number of searchers that made a detection and are in the state *"Detect"*; also, the detection rate of each of the j searchers in the state *"Search directed"* equals to $\theta^* + c_\mathrm{d}$ (Fig. 3). It can be assumed that the informational content of operator's cue is such that $c_\mathrm{d} > c_k$ for any $k = 0, \ldots, N-1$. However, this improvement in the detection rate is "local" in that it does not affect the rates of other searchers: the detection rate of searchers in the state *"Search"* changes from $\theta^* + c_k$ to $\theta^* + c_{k+1}$ only when one of the searchers detects a target and transitions to the state *"Detect"*; the detection rate $\theta^* + c_\mathrm{d}$ of searchers in the state *"Search directed"* does not change.

Such a dynamics of detection rates implies that the information provided by the operator to autonomous searchers is indeed valuable, but may be not completely accurate: the searcher is able to detect a target much faster, but the system is able to benefit from this (through an improvement of the detection rates of the remaining searchers) only after an actual detection occurs.

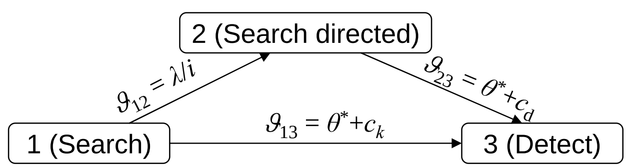

Figure 3: State diagram for autonomous searchers in the coordinated cooperative search system

Defining the states $\mathcal{S}_{ijk}$ with the corresponding state probabilities $\pi_{ijk}(t)$ in the same way as above, the Chapman-Kolmogorov equations for the described coordinated cooperative search system can be written in the form

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\pi_{ijk}(t) = & - i(\theta^* + c_k)\pi_{ijk}(t) + (i+1)(\theta^* + c_{k-1})\pi_{i+1,j,k-1}(t) \\
& - \bar{\delta}_{kN}\Big[j(\theta^* + c_\mathrm{d}) + \bar{\delta}_{i0}\lambda \Big]\pi_{ijk}(t) + \bar{\delta}_{iN}\bar{\delta}_{j0}\lambda\, \pi_{i+1,\,j-1,\,k}(t) \\
& + \bar{\delta}_{jN}\bar{\delta}_{k0}(j+1)(\theta^* + c_\mathrm{d})\,\pi_{i,\,j+1,\,k-1}(t), \quad i + j + k = N.
\end{aligned} \tag{12}
$$

Observe that the first two terms in the right-hand side of (12) represent the "unsupervised cooperation" in the system (compare to the Chapman-Kolmogorov equations (4) of the unsupervised cooperative system). The initial conditions are given by (10), and the solutions $\pi_{ijk}(t)$ do also satisfy the identity (11). Construction of an analytical solution to equations (12) is discussed in Section 3.

2.4 Monitored cooperative search

Finally, we consider a search system where after detecting a target, the autonomous searchers *engage* (e.g., attack) it, and the decision on engagement of the detected target is made autonomously or by an operator.

Namely, consider a search system of N autonomous searchers each having three possible states: *"Search"*, *"Detect"*, and *"Engage"* (see Fig. 4). Upon detecting a target, an autonomous searcher remains in the state *"Detect"* for as long as it takes to identify the target and decide whether to engage or not. Depending on the outcome of this decision, the searcher either transitions to the state *"Engage"* or returns to the state *"Search"* and resumes search. The corresponding transition rates from state *"Detect"* to states *"Engage"* and *"Search"*, ϑ_{23} and ϑ_{21}, respectively, can be intuitively defined as *decision rates* of the searcher or an operator. In general, these decision rates, as well as the detection rates of the searchers, can depend on the number i, j, k of searchers currently occupying the states *"Search"*, *"Detect"*, and *"Engage"*, correspondingly.

In the context of present work, we consider a special case where the cooperation, or information sharing within the system is based upon successful completion of the mission by a searcher, i.e., by transitioning to the state *"Engage"*. Formally, this is denoted by assuming that the detection and decision rates ϑ_{12}, ϑ_{23}, and ϑ_{21} change values due to cooperation as

$$\begin{aligned} \vartheta_{12} &= \theta^* + c_k, \\ \vartheta_{23} &= \sigma^* + c'_k, \\ \vartheta_{21} &= \rho^* + c''_k, \end{aligned} \tag{13}$$

where θ^*, σ^*, and ρ^* are the intrinsic detection and decision rates of the searcher, correspondingly, and c_k, c'_k, and c''_k are the gains in the respective rates due to cooperation.

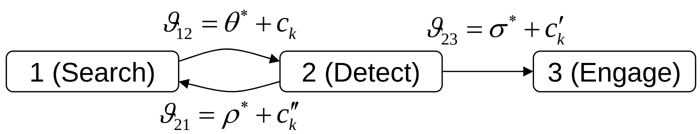

Figure 4: State diagram for autonomous searchers in the monitored cooperative search system

As before, we define $\mathcal{S}_{ijk}$ to be the state of the cooperative system with N agents, where i searchers are in the state *"Search"*, j searchers are in the state *"Detect"*, and k searchers are in the state *"Engage"*; the corresponding probability that system is in the state $\mathcal{S}_{ijk}$ at time t is $\pi_{ijk}(t)$. The differential equations governing the probabilities $\pi_{ijk}(t)$ read as

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\pi_{ijk}(t) = &- \bar{\delta}_{kN}\big[i(\theta^* + c_k) + j(\sigma^* + c'_k + \rho^* + c''_k)\big]\pi_{ijk}(t) \\ &+ \bar{\delta}_{iN}\bar{\delta}_{j0}(i+1)(\theta^* + c_k)\pi_{i+1,j-1,k}(t) \\ &+ \bar{\delta}_{i0}\bar{\delta}_{jN}(j+1)(\rho^* + c''_k)\pi_{i-1,j+1,k}(t) \\ &+ \bar{\delta}_{jN}\bar{\delta}_{k0}(j+1)(\sigma^* + c'_{k-1})\pi_{i,j+1,k-1}(t), \end{aligned} \tag{14}$$

where $i + j + k = N$. Similarly to the previously considered systems, equations (14) are equipped with initial conditions (10), and their solutions satisfy (11).

2.5 Measures of effectiveness

One of important measures of effectiveness (MOE) employed in the context of multi-agent systems operating under uncertainty is the probability of mission completion by a certain time, or, in the special case of the considered search systems, the *probability of engagement* $P_N(t)$

$$\begin{aligned} P_N(t) &= \mathsf{P}\big\{\text{search mission is completed by time } t\big\} \\ &= \mathsf{P}\big\{\text{all } N \text{ searchers have detected/engaged targets by time } t\big\}, \end{aligned} \tag{15}$$

where we will use terms "detection" or "engagement" in reference to task completion by an agent (i.e., in models of Sections 2.1–2.3 a searcher completes task by detecting a target, and in the model of Section 2.4 completion means engagement).

In many cases, a "single-number" characteristic that describes the performance of a search system is desired. Such a MOE, employed in this paper, is the *time to engage* T_β, or the time needed for all N searchers to detect/engage targets with probability $\beta \in (0, 1)$:

$$T_\beta = P_N^{-1}(\beta).$$

In our studies, the parameter β is taken to be $\beta = 0.95$, which defines $T_{0.95}$ as the time by which the search mission is completed with 95% probability.

3 NUMERICAL RESULTS

In this section we compare the performance of the models developed in Section 2 based on the time to engage $T_{0.95}$ as a measure of effectiveness. The numerical results are presented for search systems with $N = 10$ autonomous searchers.

Before proceeding to discussion of particular results obtained for each of type system presented in Sections 2.1–2.4, we comment briefly on the solution methods that were employed for dealing with the corresponding Chapman-Kolmogorov equations (4), (9), (12), (14). All of them constitute linear systems of ordinary differential equations with constant coefficients, and thus can be represented in the form

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{p}(t) = \mathbf{M}\mathbf{p}(t), \tag{16}$$

where $\mathbf{p}(t)$ is the vector of state probabilities, and $\mathbf{M}$ is a (constant) matrix. In general, analytical solutions of (16) can be obtained in closed form by, for instance, matrix exponentiation:

$$\mathbf{p}(t) = e^{\mathbf{M}t}\mathbf{p}_0, \tag{17}$$

where $\mathbf{p}_0 = \mathbf{p}(0)$ is the vector of initial conditions (initial system state probabilities). Another popular approach is based on the so-called generating function

$$G(z, t) = \sum_k p_k(t) z^k,$$

whereby the state probabilities $p_k(t)$ can be reconstructed by differentiating $G(z, t)$ with respect to z; this method is widely used in queueing theory (see, e.g., Srivastava and Kashyap, 1982).

For the governing equations (4) of the unsupervised cooperative system of Section 2.1, the corresponding form (16) is given by equations (5) where $p_k(t) = \pi_{ik}(t)$. For cooperative systems described in Sections 2.2–2.4, where each agent has three states, the corresponding form (16) of Chapman-Kolmogorov equations for state probabilities $\pi_{ijk}(t)$ is obtained by denoting

$$p_\ell(t) = \pi_{ijk}(t), \quad \ell = 0, \ldots, L, \quad \text{where} \quad L = \frac{(N+2)(N+1)}{2} - 1 = \frac{N(N+3)}{2},$$

and the index ℓ is determined by the indices i, j, and k as

$$\ell = \sum_{r=0}^{j+k-1} (r+1) + k = \frac{(j+k)(j+k+1)}{2} + k \quad \text{for all } \ 0 \le j+k \le N, \tag{18}$$

with the inverse relations given by

$$\begin{pmatrix} i \\ j \\ k \end{pmatrix} = \begin{pmatrix} N - \varpi \\ \frac{1}{2}\varpi(\varpi+3) - \ell \\ \ell - \frac{1}{2}\varpi(\varpi+1) \end{pmatrix}, \quad \text{where} \quad \varpi = \left\lfloor \frac{-1+\sqrt{1+8\ell}}{2} \right\rfloor. \tag{19}$$

The above relations between $\pi_{ijk}(t)$ and $p_\ell(t)$ can be explicitly enumerated as

$$\begin{array}{ll} p_0(t) = \pi_{N00}(t), & p_5(t) \quad = \pi_{N-2,0,2}(t), \\ p_1(t) = \pi_{N-1,1,0}(t), & \qquad\cdots \\ p_2(t) = \pi_{N-1,0,1}(t), & \qquad\cdots \\ p_3(t) = \pi_{N-2,2,0}(t), & p_{L-1}(t) = \pi_{0,1,N-1}(t), \\ p_4(t) = \pi_{N-2,1,1}(t), & p_L(t) \quad = \pi_{00N}(t). \end{array}$$

Observe that for all considered systems the initial state is given by $\pi_{N00}(0) = 1$, whence the initial conditions for the probabilities $p_\ell(t)$ as defined above can be formulated as

$$p_\ell(0) = \delta_{\ell 0}, \quad \ell = 0, \dots, L. \tag{20}$$

It is easy to see that, with the exception of the monitored cooperative system of Section 2.4, the above transformations reduce the Chapman-Kolmogorov equations (4), (9), (12) to the form (16), where the matrix $\mathbf{M}$ is lower triangular, which can significantly simplify construction of the solutions. For instance, assuming that the eigenvalues of $\mathbf{M}$ are all different, solution to the Cauchy problem (16), (20) has the form

$$p_\ell(t) = \sum_{i=0}^{\ell} a_{i\ell} e^{m_{ii} t}, \quad \ell = 0, \dots, L, \tag{21}$$

where m_{ii} are the diagonal elements (eigenvalues) of $\mathbf{M}$, and the coefficients $a_{i\ell}$ are determined recursively:

$$\begin{aligned} a_{i\ell} &= \sum_{j=i}^{\ell-1} \frac{m_{\ell j}\, a_{ij}}{m_{ii} - m_{\ell\ell}}, \quad i < \ell, \\ a_{ii} &= -\sum_{j=0}^{i-1} a_{ji}, \quad a_{00} = p_0(0) = 1. \end{aligned} \tag{22}$$

The solutions of the Chapman-Kolmogorov equations (16) for the models presented in Sections 2.1–2.4 are *state* probabilities; in what follows we will also use the *detection* probabilities $P_n(t)$

$$P_n(t) = \mathsf{P}\big\{\text{exactly } n \text{ targets are detected by time } t\big\}, \quad n = 0, \dots, N, \tag{23}$$

which can be obtained using the solution $p_\ell(t)$ as

$$P_n(t) = \sum_{\ell=0}^{L} \delta_{\ell,k(\ell)}\, p_\ell(t), \tag{24}$$

where $\delta_{\ell k}$ is the Kronecker delta, and $k = k(\ell)$ is given by (19) (in the case of the unsupervised cooperative system of Section 2.1, we simply have $k = \ell$).

The probabilities $P_n(t)$, and, in particular, the *probability of detection* $P_N(t)$, will be employed next to facilitate performance comparisons between different types of cooperative systems, provided that they are of the same size N; note that the state probabilities (e.g., $\pi_{ijk}(t)$) cannot be used for this purpose directly as they are system-specific.

3.1 Unsupervised cooperative search system

In Jeffcoat et al. (2006) it was shown that in the case when cooperation has no effect on detection capabilities of the searchers, $c_k = 0$, the probability of detection is equal to the probability that all N searchers detect targets independently (see also (2))

$$P_N^*(t) = \left(1 - e^{-\theta^* t}\right)^N.$$

To compare the unsupervised cooperative model with the system of non-cooperative (independent) searchers, we assume that the detection rates (3) in the unsupervised cooperative system satisfy (see Fig. 5):

$$\begin{aligned} &\theta^* = 1, \\ &c_k = \varkappa \tfrac{2}{\pi} \arctan \tfrac{k}{2}, \quad k = 0, \dots, N-1, \quad \text{where} \quad \varkappa = 1/2. \end{aligned} \tag{25}$$

Expression (25) implies that while each next detection improves the search rate θ_k of the cooperative searchers that have not yet made a detection, the marginal improvement in the search rate θ_k is decreasing with time. Also observe that the parameter $\varkappa$ in (25) represents the upper bound for detection rate gains c_k due to cooperation, and, moreover, this upper bound is not attainable:

$$c_k < \varkappa, \quad k = 0, \dots, N-1.$$

In particular, the time to engage $T_{0.95} = P_{10}^{-1}(0.95)$ for the unsupervised cooperative search system equals to

$$T_{0.95} = 3.797,$$

a 28% improvement over the corresponding value ($T_{0.95} = 5.275$) for a system of 10 non-coooperative (independent) searchers with detection rates $\theta = 1$.

3.2 Coordinated search system

For the coordinated search system described in Section 2.2, we assume that the search rate of the supervisor is equal to the intrinsic search rate of autonomous searchers, and receipt of operator's cue boosts the search rate of the autonomous searcher by the amount $\varkappa$:

$$\lambda = \theta^* = 1, \quad c_{\mathrm{d}} = \varkappa = 1/2. \tag{26}$$

The detection probabilities $P_n(t)$ are computable via the state probabilities $\pi_{ijk}(t)$, which solve the Chapman-Kolmogorov equations (9), as

$$P_n(t) = \sum_{\substack{k=n \\ i+j+k=N \\ i,j \geq 0}} \pi_{ijk}(t), \quad n = 0, \dots, N, \tag{27}$$

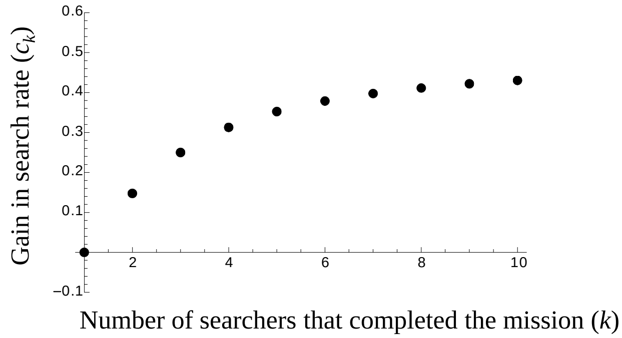

Figure 5: Dynamics of cooperative gains c_k in detection rates and decision rates in search systems with cooperation

or, equivalently, by (24). The time to engage for the the coordinated search system equals to

$$T_{0.95} = 4.329,$$

which is a 14% deterioration comparing to the unsupervised cooperative system.

3.3 Coordinated cooperative search system

In the coordinated cooperative search model, we assumed that the detection rates improve due to cooperation in accordance to (25); the values for operator's search rate λ and operator-cued rate $\theta^* + c_\mathrm{d}$ are given by (26). The detection probabilities $P_n(t)$ are calculated using the solutions $\pi_{ijk}(t)$ of Chapman-Kolmogorov equations (12) in the same way as in (27), and the time to engage in coordinated cooperative system has the value of

$$T_{0.95} = 2.857,$$

a 34% improvement over the coordinated search model and 24% improvement over the unsupervised cooperative search model.

3.4 Monitored cooperative search system

In the context of the monitored cooperative search system described in Section 2.4, it is assumed that cooperation increases the rates $\vartheta_{12} = \theta^* + c_k$ and $\vartheta_{23} = \sigma^* + c'_k$, and decreases the rejection rate $\vartheta_{21} = \rho^* + c''_k$. This means that cooperation not only improves the search capabilities of the autonomous agents, but also improves the target assessment process by the supervisor/operator (for instance, improvements in search capabilities lead to more reliable target detection, so that less time is needed to identify a detected object as a target, which also reduces rejection rates ϑ_{21} due to fewer false target detections). In particular, we assume that the search rates improve due to cooperation in accordance to (25); for simplicity, similar dynamics is assumed with respect to the decision rates $\sigma^* + c'_k$ and $\rho^* + c''_k$:

$$\begin{aligned} &\sigma^* = \rho^* = 1, \\ &c'_k = \varkappa \tfrac{2}{\pi} \arctan \tfrac{k}{2}, \quad c''_k = -\varkappa \tfrac{2}{\pi} \arctan \tfrac{k}{2}, \end{aligned} \tag{28}$$

where, as before, the value of $\varkappa$ is taken at $\varkappa = 1/2$. The corresponding time to engage is

$$T_{0.95} = 7.859.$$

Clearly, such an increase in $T_{0.95}$ in comparison to the models considered above can be attributed to the additional delays associated with supervisor's decision making in state *"Detect"*, as well as to the possibility of cycling between the states *"Search"* and *"Detect"*.

On the other hand, this example illustrates that the time to engage $T_{0.95}$, and generally, the probability of engagement $P_N(t)$, while serving well as measures of effectiveness, may not always be appropriate for measuring the degree of cooperativeness in a given system.

4 MEASURING THE DEGREE OF COOPERATION

The introduced measures of effectiveness $P_N(t)$ and $T_{0.95} = P_N^{-1}(0.95)$ allow for quantifying how fast all N searchers accomplish the mission. However, they do not shed light on how the cooperative system evolves, or progresses, *during* the mission. In this section, we attempt to answer the following question: given a cooperative[2] system, how can one estimate its "degree of cooperativeness," or, in other words, how can one distinguish between "more cooperative" and "less cooperative" systems?

Intuitively, the proposed approach to measuring cooperation in multi-agent systems can be described as follows: for a given cooperative system, consider its non-cooperative variant, where the agents are not allowed to cooperate, and determine by how much the capabilities (e.g., detection rates) of the non-cooperative agents must be improved so that the non-cooperative system performs as well as the cooperative one.

In order to introduce our approach formally, it is convenient to adopt the following notation. For a given multi-agent cooperative system where the individual members are modeled using Markov chains as presented above, let $\boldsymbol{\vartheta}$ denote the vector of transition rates of a cooperative agent. Per above, this vector can be represented in the form

$$\boldsymbol{\vartheta} = \boldsymbol{\theta}^* + \mathbf{c}, \tag{29}$$

where $\boldsymbol{\theta}^*$ are the intrinsic values of the components of $\boldsymbol{\vartheta}$, and $\mathbf{c}$ are the corresponding gains (or reductions) due to cooperative effects; note that both $\boldsymbol{\theta}^*$ and $\mathbf{c}$ may contain zero elements.

Without loss of generality, it is assumed that cooperation effects in the system lead to either an increase or decrease of a given parameter (transition rate) of an agent. With respect to this, we present the vector $\boldsymbol{\vartheta}$ as $\boldsymbol{\vartheta} = (\boldsymbol{\vartheta}_+, \boldsymbol{\vartheta}_-)$, where $\boldsymbol{\vartheta}_+$ corresponds to the transition rates that are non-decreasing due to cooperation effects, and $\boldsymbol{\vartheta}_-$ stands for the parameters (transition rates) of a searcher that decrease due to cooperation. In view of this, it is convenient to rewrite (29) as

$$\boldsymbol{\vartheta} = \big(\boldsymbol{\theta}_+^* + \mathbf{c}_+,\ \boldsymbol{\theta}_-^* + \mathbf{c}_-\big), \tag{30}$$

where $\mathbf{c}_+ \geq \mathbf{0}$ and $\mathbf{c}_- < \mathbf{0}$.

In addition, without loss of generality we assume that the magnitudes of gains $\mathbf{c}_\pm$ due to cooperative effects do not exceed the corresponding intrinsic values:

$$|\mathbf{c}_\pm| \leq |\boldsymbol{\theta}_\pm^*|. \tag{31}$$

To formalize the outlined above approach to measuring cooperation in distributed system, its three key steps need to be determined: (i) how the "non-cooperative" system is defined, (ii) how the "improvement" in the capabilities of non-cooperative agents is defined, and (iii) how the "performances" of cooperative and non-cooperative systems are compared. Below we discuss each of these steps.

[2]The term *cooperative* is understood here in a broad sense, not restricted to the context of Section 2.1.

4.1 Equivalent independent systems

Our approach to measuring the degree of cooperativeness in a given system is based on evaluating how close its performance matches that of an *equivalent independent system* (EIS). The EIS is obtained from a given cooperative system by disregarding the parameters (e.g., detection rate gains) that are attributed to cooperation within the system. Formally, this is accomplished by letting $\mathbf{c} = \mathbf{0}$ in (29). With respect to the cooperative search systems described in Sections 2.1–2.4, this entails discarding the non-starred terms in the state transition rates.

For example, the EIS that corresponds to the unsupervised cooperative system of Section 2.1 is obtained by setting $c_k = 0$ for all $k = 0, \ldots, N-1$; in other words, the EIS is comprised of agents described by the state transition diagram in Fig. 1 whose detection rates are fixed at the intrinsic value θ^*.

Since interaction (cooperation) within the coordinated search system of Section 2.2 is facilitated through intermediacy of the supervisor that distributes additional information to the searchers at a rate λ/i, the corresponding equivalent independent system is obtained by setting $\lambda = 0$ and $c_\mathrm{d} = 0$. Observe that this corresponds to removing the state *"Search directed"*, whereby the EIS is comprised of agents that are also described by state diagram in Fig. 1 with detection rate θ^*.

The EIS for the coordinated cooperative model of Section 2.3 is similarly obtained by setting $c_k = 0$, $k = 0, \ldots, N-1$, $\lambda = 0$, and $c_\mathrm{d} = 0$, so that the corresponding EIS is the same as in the above two cases.

Finally, the EIS of the monitored cooperative system of Section 2.4 is obtained by setting $c_k = 0$, which implies that EIS agents are described by the state transition diagram in Fig. 4 with the corresponding transition rates fixed at θ^*, σ^*, and ρ^*. Note that, unlike the EIS for coordinated and coordinated cooperative systems, the EIS for the monitored cooperative search system must retain the state *"Detect"* associated with supervisor's actions (i.e., approval of a detected target), because the primary goal of these actions is not to improve system's performance via cooperation.

4.2 Coefficient of cooperation and similarity measures

Having defined the EIS, we quantify the degree of cooperation in a distributed multi-agent system by introducing the *coefficient of cooperation*, α, as a factor by which the parameters (e.g., detection rates) of the EIS agents must be increased so that the performance of the EIS matches closely the performance of the cooperative system in question.

Using the notation (30) and the assumption (31), the vector of parameters (transition rates) of the EIS members is chosen as[3]

$$\boldsymbol{\vartheta}^*_\alpha = \big((1+\alpha)\boldsymbol{\theta}^*_+,\ (1-\alpha)\boldsymbol{\theta}^*_-\big), \tag{32}$$

where the coefficient of cooperation $\alpha \in [0, 1]$ has to be adjusted in such a way that the performance of the equivalent independent system is close to that of the cooperative system under consideration, whose agents are characterized by the transition rates $\boldsymbol{\vartheta} = (\boldsymbol{\theta}^*_+ + \mathbf{c}_+, \boldsymbol{\theta}^*_- + \mathbf{c}_-)$. Note that expression (32) implies that all the intrinsic values $\boldsymbol{\theta}^*_\pm$ are increased/decreased by the same proportion α so as to "match" the corresponding cooperative gains $\mathbf{c}_\pm$, which may not necessarily be of the same magnitude (i.e., gains in detection rates due to cooperation may be much higher than gains in decision rates, etc.) While this can be accommodated by introducing multiple α_i, in the scope of the present endeavor we are interested in the coefficient of cooperation as a single number that would characterize cooperation in the given system.

[3] In certain cases, it may be of interest to determine the coefficient of cooperation by applying the $(1 \pm \alpha)$ factors to only select elements of $\boldsymbol{\vartheta}$; thus, a general expression for (32) can be written as $\boldsymbol{\vartheta}^*_\alpha = \big((1+\alpha)\boldsymbol{\theta}^*_+,\ (1-\alpha)\boldsymbol{\theta}^*_-,\ \boldsymbol{\theta}^*_0\big)$, where $\boldsymbol{\theta}^*_0$ denotes the elements of $\boldsymbol{\vartheta}^*_\alpha$ that are not supposed to determine directly the coefficient of cooperation α.

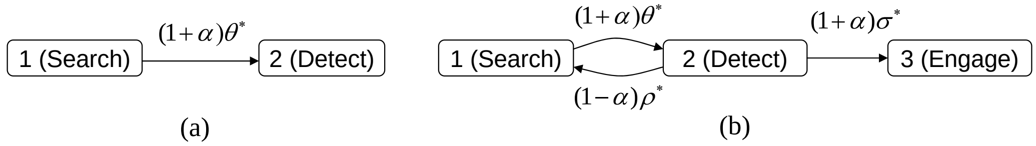

Figure 6: State diagrams for autonomous agents in the equivalent independent system that correspond to: (a) the unsupervised cooperative, coordinated, and coordinated cooperative search systems, and (b) the monitored cooperative search system.

By applying (32) to the described above equivalent independent systems that correspond to cooperative search systems of Sections 2.1–2.4, we obtain the representations for EIS agents as shown in Fig. 6.

As it has been stated, the coefficient of cooperation has to be selected such that the cooperative system and its corresponding independent equivalent system exhibit "similar" performance. In the presented framework, the state of a multi-agent system at any given time moment is characterized by a probability distribution governing the corresponding Markov process. Thus, the problem of quantifying the similarity between two multi-agent systems in the given context effectively reduces to measuring the distance between two non-stationary probability distributions. In general, the methods of quantifying the dissimilarities between probability distributions are application-dependent, and in scope of this study we employ two approaches. The first measure of distance between distributions is the well-known Kullback-Leibler (KL) divergence, or relative entropy (Kullback and Leibler, 1951; Kullback, 1968): given two (discrete) probability distributions $h_1(\mathbf{x})$ and $h_2(\mathbf{x})$ defined over the same set of realizations $\mathbf{X}$, the symmetric KL divergence is defined as

$$J_{12} = \sum_{\mathbf{x}\in\mathbf{X}} (h_1(\mathbf{x}) - h_2(\mathbf{x})) \ln \frac{h_1(\mathbf{x})}{h_2(\mathbf{x})}. \tag{33}$$

The KL divergence possesses a number of useful properties, such as non-negativity: $J_{12} \geq 0$ with $J_{12} = 0$ only in the case when the distributions h_1 and h_2 are identical, symmetry: $J_{12} = J_{21}$, and so on; see Kullback (1968) for details. The KL divergence is widely used in information theory, where it serves as a proxy for the average information that discriminates distributions h_1 and h_2.

The other measure of distance between distributions that we will use to judge the degree of cooperation in a distributed system, is the 1-norm, or "manhattan" pointwise distance between distributions:

$$d_{12} = \left\| h_1(x) - h_2(\mathbf{x}) \right\|_1 = \sum_{\mathbf{x}\in\mathbf{X}} \left| h_1(\mathbf{x}) - h_2(\mathbf{x}) \right|. \tag{34}$$

The above generic definitions of distance between probability distributions need to be modified appropriately in order to be applicable in the scope of the present study, where the degree of similarity between *non-stationary* probability distributions, i.e., distributions that change in time, is of interest.

Given that the cooperative system in question and its corresponding equivalent independent systems have the same number N of agents, it is appropriate to associate the performance of each system with the number $n = 0, \ldots, N$, of agents that have completed the mission by time t. Thus, the similarity between the EIS and the cooperative systems can be quantified by measuring the distance between the detection probabilities $P_n(t)$ (24) of the cooperative system and the analogous quantities $P_n^*(t)$ of the equivalent independent system; note that for each $t \geq 0$, the detection probabilities form a probability distribution:

$$\sum_{n=0}^{N} P_n(t) = \sum_{n=0}^{N} P_n^*(t) = 1, \quad t \geq 0.$$

The detection probabilities $P_n(t)$ of a cooperative system are obtained from the corresponding Markov process state probabilities using (24), and the probabilities $P_n^*(t)$ of the equivalent independent system are given by the binomial distribution

$$P_n^*(t) = P_n^*(t,\alpha) = \binom{N}{n}\left[f(t;\boldsymbol{\vartheta}_\alpha^*)\right]^n\left[1 - f(t;\boldsymbol{\vartheta}_\alpha^*)\right]^{N-n}, \quad n = 0,\dots,N, \tag{35}$$

where $f(t;\boldsymbol{\vartheta}_\alpha^*)$ is the probability that an agent of the equivalent independent system completes its task by time t.

Then, as measures of distance between the non-stationary distributions $P_n(t)$ and $P_n^*(t,\alpha)$ we will use the maximum KL divergence:

$$J_{12}(\alpha) = \max_{t\geq 0}\sum_{n=0}^{N}\left[P_n(t) - P_n^*(t,\alpha)\right]\ln\frac{P_n(t)}{P_n^*(t,\alpha)}, \tag{36}$$

and the maximum “manhattan”, or 1-norm distance

$$d_{12}(\alpha) = \max_{t\geq 0}\sum_{n=0}^{N}\left|P_n(t) - P_n^*(t,\alpha)\right| \tag{37}$$

Observe that the defined in such a way KL divergence and 1-norm manhattan distance are finite and therefore well-defined for any $t \geq 0$, due to the fact that

$$P_n(0) = P_n^*(0,\alpha) = \delta_{0n} \quad \text{and} \quad \lim_{t\to\infty} P_n(t) = \lim_{t\to\infty} P_n^*(t,\alpha) = \delta_{Nn},$$

(i.e, 0 agents have their tasks completed at $t = 0$, and all N agents complete their tasks with probability 1 as $t \to \infty$), and therefore the corresponding sums in (36) and (37) vanish at $t = 0$ and $t \to \infty$.

Then, the coefficient of cooperation α^* is chosen as the value of α that minimizes the corresponding similarity measure between distributions:

$$\min_{\alpha\in[0,1]} J_{12}(\alpha) = \min_{\alpha\in[0,1]}\max_{t\geq 0}\sum_{n=0}^{N}\left[P_n(t) - P_n^*(t,\alpha)\right]\ln\frac{P_n(t)}{P_n^*(t,\alpha)}, \tag{38}$$

or

$$\min_{\alpha\in[0,1]} d_{12}(\alpha) = \min_{\alpha\in[0,1]}\max_{t\geq 0}\sum_{n=0}^{N}\left|P_n(t) - P_n^*(t,\alpha)\right|. \tag{39}$$

In the next subsection we report computational results on determining the coefficient of cooperation for the cooperative search models presented in Sections 2.1–2.4.

4.3 Numerical results and discussion

Here we report the numerical results on computing the coefficient of cooperation for the cooperative search systems developed in Section 2 with $N = 10$ searchers and detection and decision rates given by (25), (26), and (28). As explained above, the equivalent independent systems (or, more precisely, their independent

Table 1: Coefficient of cooperation and the corresponding similarity measures

	Manhattan distance, d_{12}		KL divergence, J_{12}	
Type of system	Coefficient of cooperation, α^*	Similarity $d_{12}(\alpha^*)$	Coefficient of cooperation, α^*	Similarity $J_{12}(\alpha^*)$
Unsupervised cooperative	0.258	0.16898	0.249	4.083×10^{-2}
Coordinated	0.084	0.10999	0.089	2.104×10^{-2}
Coordinated cooperative	0.451	0.29992	0.443	1.286×10^{-1}
Monitored cooperative	0.236	0.32603	0.230	1.444×10^{-1}

agents) are described by Markov chain diagrams in Fig. 6, where, as before, we used the following intrinsic values of the corresponding detection and decision rates:

$$\theta^* = \sigma^* = \rho^* = 1.$$

Then, it is easy to see that the probability that an agent in Fig. 6(a) completes its task (transitions to the state *"Detect"*) at time t is exponentially distributed:

$$f(t, \boldsymbol{\vartheta}^*_\alpha) = 1 - e^{-(1+\alpha)t}. \tag{40}$$

Also, it can be shown that the probability that an agent of the equivalent independent system corresponding to the monitored cooperative search model (Fig. 6(b)) engages a target at time t is given by

$$\begin{aligned} f(t, \boldsymbol{\vartheta}^*_\alpha) = 1 - \frac{1}{2\Delta_\alpha^2}\Big\{&\big[5 - 3\Delta_\alpha - \alpha(2 + 3\alpha + \Delta_\alpha)\big]e^{-\frac{1}{2}(3+\alpha+\Delta_\alpha)t} \\ &+ \big[5 + 3\Delta_\alpha + \alpha(-2 - 3\alpha + \Delta_\alpha)\big]e^{-\frac{1}{2}(3+\alpha-\Delta_\alpha)t}\Big\}, \end{aligned} \tag{41}$$

where $\Delta_\alpha = \sqrt{(1-\alpha)(3\alpha + 5)}$.

The obtained values of the coefficient of cooperation α^* and the corresponding similarity measures $d_{12}(\alpha^*)$ and $J_{12}(\alpha^*)$ (38)–(39) between the detection probabilities $P_n(t)$ of the cooperative systems and the detection probabilities $P_n^*(t, \alpha^*)$ (35) are shown in Table 1.

First, we note that the two employed metrics for measuring similarities between non-stationary probability distributions, the Kullback-Leibler divergence (36) and the "manhattan" 1-norm distance (37) produce similar results in terms of the coefficient of cooperation α^* as an optimal solution of (38) and (39), respectively, and the corresponding minimal distances $J_{12}(\alpha^*)$ and $d_{12}(\alpha^*)$ between distributions.

The results for the unsupervised cooperative system indicate that the searchers in the corresponding equivalent independent search system must possess detection rates about 25% times higher than the intrinsic search rate θ^* of the cooperative searchers in order to match their performance throughout the mission. As to be expected, the coordinated cooperative system exhibits a higher degree of cooperation, which is evidenced by the coefficient of cooperation that is almost twice as high at about 45%. In addition, the coordinated cooperative system is "less similar" to the independent system than the unsupervised cooperative system, which is manifested by a higher value similarity measure, especially in the case when KL divergence is used as a metric of distance between distributions.

An interesting observation can be made regarding the coordinated search system, where the supervisor/coordinator can boost the detection rate of an individual searcher by 50% ($c_{\mathrm{d}} = 1/2$), which exceeds the cooperative gains in detection rate in the unsupervised cooperative system. Our computational results imply that independent searchers must have detection rates about 8–9% higher than the intrinsic detection rates θ^* of searchers in the coordinated system in order to perform as well as the coordinated system. Thus, it can be said that the coordinated system is significantly less "cooperative" than the unsupervised search system of Section 2.1.

To explain this conclusion, recall that unlike the other considered systems, the coordinated search system does not allow for a "global" rate improvement among the searchers; the rate improvements $\theta^* \to \theta^* + c_{\mathrm{d}}$ of the searchers that received a directional cue from the supervisor are localized in time and bear no effect on the detection rates of other searchers. To put it differently, the searchers in the coordinated system perform their search independently, and occasionally may receive "gift" cues from the operator; the new information contained in these cues, however, is not shared within the system, and thus does not lead to improvement of capabilities of other members of the system.

Also, it is of interest to note that similarity measures $d_{12}(\alpha^*)$, $J_{12}(\alpha^*)$ for the coordinated system are the lowest among all four considered systems, i.e., the coordinated system can be fitted by an independent systems most closely.

With regard to the monitored cooperative system, recall that its time to engage $T_{0.95}$ as a measure of system's effectiveness was inferior to all other systems. The results reported in Table 1, nevertheless, indicate that the monitored system, despite the possibilities of "resetting" the search due to the "backward" transition from the state *"Detect"* to *"Search"*, possesses a high degree of cooperativeness, as its coefficient of cooperation at about 23% is almost the same as that of the unsupervised cooperative system. Also, the corresponding values of $d_{12}(\alpha^*)$ and $J_{12}(\alpha^*)$ suggest that the monitored cooperative system possesses the lowest degree of "similarity" to the corresponding equivalent independent system.

CONCLUSIONS

We presented an approach to modeling and analysis of distributed multi-agent search systems where the autonomous agents may cooperate among each other, and/or with supervisor/operator, in order to achieve the system's objective. It was assumed that the cooperation is facilitated by means of information sharing among the autonomous agents and/or the operator, with the purpose of improving the effectiveness of the autonomous agents. The evolution of cooperative systems was modeled using discrete-state, continuous-time Markov processes. We introduced a technique for measuring and quantification of cooperation effects within such systems that is based on minimization of Kullback-Leibler or pointwise 1-norm distances between non-stationary probability distributions, which characterize the cooperative system and its equivalent independent system. The introduced measure of cooperativeness, the coefficient of cooperation, is illustrated on four different types of distributed search systems: an unsupervised cooperative, coordinated, coordinated cooperative, and monitored cooperative search systems.

The proposed approach to modeling of distributed cooperative systems that operate in uncertain environments is capable of yielding valuable quantitative and qualitative insights into the fundamental properties of the described cooperative systems, as well as serving as a benchmark for more elaborate simulation-based models. Future research directions include development of general models for cooperative systems where an agent is described by a Markov process on a general graph with more than one absorbing state, as well as developing methods for optimization of such systems.

AUTHOR STATEMENT

The views expressed in this paper are those of the authors and do not reflect the official policy or position of the Air Force Research Laboratory, Department of Defense or the US Government.

ACKNOWLEDGEMENTS

This work was partially supported by AFOSR grants FA8651-07-1-0001 and LRIR 07MN01COR.

References

Brémaud, P. (1999) *Markov chains: Gibbs fields, Monte Carlo simulaiton, and queues*, Springer, New York.

Butenko, S., Murphey, R., and Pardalos, P. (Eds.) (2003) *Cooperative Control: Models, Applications and Algorithms*, Springer.

Grundel, D., Murphey, R., and Pardalos, P. (Eds.) (2004) *Theory and Algorithms for Cooperative Systems*, volume 4 of *Series on Computers and Operations Research*, World Scientific.

Hirsch, M. J., Commander, C. W., Pardalos, P. M., and Murphey, R. (Eds.) (2009) *Optimization and Cooperative Control Strategies*, volume 381 of *Lecture Notes in Control and Information Sciences*, Springer.

Hirsch, M. J., Pardalos, P. M., Murphey, R., and Grundel, D. (Eds.) (2007) *Advances in Cooperative Control and Optimization*, volume 369 of *Lecture Notes in Control and Information Sciences*, Springer.

Jeffcoat, D., Krokhmal, P., and Zhupanska, O. (2006) "Effects of cueing in cooperative search," *Naval Research Logistics*, **53** (8), 814–821.

Jeffcoat, D., Krokhmal, P., and Zhupanska, O. (2007) "Markov Chain Approach to Analysis of Cooperation in Multi-Agent Search Missions," in: D. Grundel, R. Murphey, P. Pardalos, and O. Prokopyev (Eds.) "Cooperative Systems: Control and Optimization," volume 588 of *Springer Lecture Notes in Economics and Mathematical Sciences*, 171–184.

Kullback, S. (1968) *Information Theory and Statistics*, Dover, Gloucester, MA.

Kullback, S. and Leibler, R. A. (1951) "On Information and Sufficiency," *The Annals of Mathematical Statistics*, **22** (1), 79–86.

Papoulis, A. and Pillai, S. U. (2002) *Probability, Random Variables, and Stochastic Processes*, McGraw Hill, New York, 4th edition.

Peleg, B. and Sudhölter, P. (2007) *Introduction to the Theory of Cooperative Games*, Springer, Berlin.

Srivastava, H. M. and Kashyap, B. R. (1982) *Special Functions in Queuing Theory and Related Stochastic Processes*, Academic Press, New York.