CSc 144 — Discrete Structures for Computer Science I
Spring 2023 (McCann)

Collected Definitions for Exam #2

I can't recall the last time I didn't ask a definition question on an exam. To help you better prepare yourself for such questions, I've assembled this list. My pledge to you: If I ask you for a definition on the exam, the term will come from this list. Note that this is not a complete list of the definitions given in class. You should know the others, too, but I won't specifically ask you for their definitions on the exam.

Once in a while a student will express disappointment that I ask definition questions on exams. My justification is that I think it's important for you to know what the core terms mean so that you can use them correctly and effectively. At the same time, I don't require that you memorize the exact wording of the definitions you see here. If you provide a definition in your own words that captures all of the detail found here, without adding anything incorrect, that's fine.

Topic 4: Arguments

- "An *argument* is a connected series of statements to establish a definite proposition." [Credit: Monty Python's Flying Circus, Series 3, Episode 3 ("The Money Programme"), "Argument Clinic."]
- An argument that moves from specific observations to a general conclusion is an *inductive argument*.
- An argument that uses accepted general principles to explain a specific situation is a *deductive argument*.
- Any deductive argument of the form $(p_1 \wedge p_2 \wedge \ldots \wedge p_n) \to q$ is *valid* if the conclusion must follow from the hypotheses.
- A valid argument that also has true hypotheses is a *sound* argument.
- A *fallacy* is an argument constructed with an improper inference.

Topic 5: Direct Proofs of $p \to q$

- A *conjecture* is a statement with an unknown truth value.
- A *theorem* is a conjecture that has been shown to be true.
- A sound argument that establishes the truth of a theorem is a *proof*.
- A *lemma* is a simple theorem whose truth is used to construct more complex theorems.
- A *corollary* is a theorem whose truth follows directly from another theorem.

(Continued . . .)

Topic 6: Additional Set Concepts

- A *set* is an unordered collection of unique objects.
- Set A is a *subset* of set B (written $A \subseteq B$) if every member of A can be found in B.
- A is a *proper subset* of B (written $A \subset B$) if $A \subseteq B$ and $A \neq B$.
- The *power set* of a set A (written $\mathcal{P}(A)$) is the set of all of A's subsets, including the empty set.
- Two sets are *disjoint* if their intersection is $\emptyset$ (the empty set).
- A *partition* of a set separates all of its members into disjoint subsets.
- An *ordered pair* is a group of two items (a, b) such that $(a, b) \neq (b, a)$ unless $a = b$.
- The *Cartesian Product* of two sets A and B (written $A \times B$) is the set of all ordered pairs (a, b) such that $a \in A$ and $b \in B$.

Topic 7: Relations

- A *(binary) relation* from set X to set Y is a subset of the Cartesian Product of the domain X and the codomain Y.
- A relation R on set A is *reflexive* if $(a, a) \in R$, $\forall a \in A$.
- A relation R on set A is *symmetric* if, whenever $(a, b) \in R$, then $(b, a) \in R$, for $a, b \in A$.
- A relation R on set A is *antisymmetric* if $(x, y) \in R$ and $x \neq y$, then $(y, x) \notin R$, $\forall x, y \in A$.
- A relation R on set A is *transitive* if, whenever $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$, for $a, b, c \in A$.
- The *inverse* of a relation R on set A, denoted R^{-1}, contains all of the ordered pairs of R with their components exchanged. (That is, $R^{-1} = \{(b, a) \mid (a, b) \in R\}$.)
- Let G be a relation from set A to set B, and let F be a relation from B to set C. The *composite* of F and G, denoted $F \circ G$, is the relation of ordered pairs $(a, c), a \in A, c \in C$, such that $b \in B$, $(a, b) \in G$, and $(b, c) \in F$.
- A relation R on set A is an *equivalence relation* if it is reflexive, symmetric, and transitive.
- The *equivalence class* of an equivalence relation R on set B, and an element $b \in B$, is $\{c \mid c \in B \wedge (b, c) \in R\}$ and is denoted $[b]$.
- A relation R on set A is a *(reflexive/weak) partial order* if it is reflexive, <u>anti</u>symmetric, and transitive.
- A relation R on set A is *irreflexive* if, for all members of A, $(a, a) \notin R$.
- A relation R on set A is an *irreflexive* (or *strict*) *partial order* if it is <u>ir</u>reflexive, antisymmetric, and transitive.
- Let R be a weak partial order on set A. a and b are said to be *comparable* if $a, b \in A$ and either $a \preceq b$ or $b \preceq a$ (that is, either $(a, b) \in R$ or $(b, a) \in R$).
- A weak partially-ordered relation R on set A is a *total order* if every pair of elements $a, b \in A$ are comparable.