CSc 144 — Discrete Structures for Computer Science I
Spring 2023 (McCann)

Collected Definitions for Exam #3

This is the 'official' collection of need-to-know definitions for Exam #3. I can't recall the last time I didn't ask a definition question on an exam. To help you better prepare yourself for definition questions, I've assembled this list. My pledge to you: If I ask you for a definition on the exam, the term will come from this list. Note that this is not a complete list of the definitions given in class. You should know the others, too, but I won't specifically ask you for their definitions on the exam.

Topic 8: Functions

- A *function* from set X to set Y, denoted $f : X \to Y$, is a relation from X to Y such that $f(x)$ is defined $\forall x \in X$ and, for each $x \in X$, there is exactly one $(x, y) \in f$.
- For each of the following, let $f : X \to Y$ be a function, and assume $f(n) = p$.
  - X is the *domain* of f; Y is the *codomain* of f.
  - f *maps* X to Y.
  - p is the *image* of n; n is the *pre-image* of p.
  - The *range* of f is the set of all images of elements of X. (Note that the range need not equal the codomain.)
- The *floor* of a value n, denoted $\lfloor n \rfloor$, is the largest integer $\leq n$.
- The *ceiling* of a value m, denoted $\lceil m \rceil$, is the smallest integer $\geq m$.
- A function $f : X \to Y$ is *injective* (a.k.a. *one-to-one*) if, for each $y \in Y$, $f(x) = y$ for at most one member of X.
- A function $f : X \to Y$ is *surjective* (a.k.a. *onto*) if f's range is Y (the range = the codomain).
- A *bijective* function (a.k.a. a *one-to-one correspondence*) is both injective and surjective.
- The *inverse* of a bijective function f, denoted f^{-1}, is the relation $\{(y, x) \mid (x, y) \in f\}$.
- Let $f : Y \to Z$ and $g : X \to Y$. The *composition* of f and g, denoted $f \circ g$, is the function $h = f(g(x))$, where $h : X \to Z$.
- A function $f : X \times Y \to Z$ (or $f(x, y) = z$) is a *binary* function.

Topic 9: Indirect ("Contra") Proofs of $p \to q$

No new definitions in this topic!

Topic 10: Properties of Integers

- Let i and j be positive integers. j is a *factor* of i when $i\%j = 0$.
- A positive integer p is *prime* if $p \geq 2$ and the only factors of p are 1 and p.
- A positive integer p is *composite* if $p \geq 2$ and p is not prime.
- Let x and y be integers such that $x \neq 0$ and $y \neq 0$. The *Greatest Common Divisor* (GCD) of x and y is the largest integer i such that $i \mid x$ and $i \mid y$. That is, `gcd(x,y) = i`.
- If the GCD of a and b is 1, then a and b are *relatively prime*.
- When the members of a set of integers are all relatively prime to one another, they are *pairwise relatively prime*.
- Let x and y be positive integers. The *Least Common Multiple* (LCM) of x and y is the smallest integer s such that $x \mid s$ and $y \mid s$. That is, `lcm(x,y) = s`.

(Continued ...)

Topic 11: Sequences and Strings

- A *sequence* is the ordered range of a function from a set of integers to a set S.
- In an *arithmetic sequence* (a.k.a. *arithmetic progression*) a, $a_{n+1} - a_n$ is constant. This constant is called the *common difference* of the sequence.
- In a *geometric sequence* (a.k.a. *geometric progression*) g, $\frac{g_{n+1}}{g_n}$ is constant. This constant is called the *common ratio* of the sequence.
- An *increasing* (a.k.a. *non-decreasing*) sequence i is ordered such that $i_n \leq i_{n+1}$.
- A *strictly increasing* sequence i is ordered such that $i_n < i_{n+1}$.
- A *non-increasing* (a.k.a. *decreasing*) sequence i is ordered such that $i_n \geq i_{n+1}$.
- A *strictly decreasing* sequence i is ordered such that $i_n > i_{n+1}$.
- Sequence x is a *subsequence* of sequence y when the elements of x are found within y in the same relative order.
- A *string* is a contiguous finite sequence of zero or more elements drawn from a set called the *alphabet*.
- A set is *finite* if there exists a bijective mapping between it and a set of cardinality n, $n \in \mathbb{Z}^*$.
- A set is *countably infinite* (a.k.a. *denumerably infinite*) if there exists a bijective mapping between the set and either $\mathbb{Z}^*$ or $\mathbb{Z}^+$.
- A set is *countable* if it is either finite or countably infinite. If neither, the set is *uncountable*.

Topic 12: Counting *(through Princ. of I/E)*

- The *(Generalized) Pigeonhole Principle*: If n items are placed in k boxes, then at least one box contains at least $\lceil \frac{n}{k} \rceil$ items.
- The *Multiplication Principle* (a.k.a. the *Product Rule*): If there are s steps in an activity, with n_1 ways of accomplishing the first step, n_2 of accomplishing the second, etc., and n_s ways of accomplishing the last step, then there are $n_1 \cdot n_2 \cdot \ldots \cdot n_s$ ways to complete all s steps.
- The *Addition Principle* (a.k.a. the *Sum Rule*): If there are t tasks, with n_1 ways of accomplishing the first, n_2 ways of accomplishing the second, etc., and n_t ways of accomplishing the last, then there are $n_1 + n_2 + \ldots + n_t$ ways to complete one of these tasks, assuming that no two tasks interfere with one another.
- The *Principle of Inclusion-Exclusion for Two Sets* says that the cardinality of the union of sets M and N is the sum of their individual cardinalities excluding the cardinality of their intersection. That is:
  $|M \cup N| = |M| + |N| - |M \cap N|$
- The *Principle of Inclusion-Exclusion for Three Sets* says that the cardinality of the union of sets M, N, and O is the sum of their individual cardinalities excluding the sum of the cardinalities of their pairwise intersections and including the cardinality of their intersection. That is:
  $|M \cup N \cup O| = |M| + |N| + |O| - (|M \cap N| + |M \cap O| + |N \cap O|) + |M \cap N \cap O|$

There are more definitions in Topic 12, but they define terms that aren't in the material covered by Exam #3.