# Topic 2:

Logic



# What Is Logic?

**Definition: Philosophical Logic**

**Definition: Mathematical Logic**

# Propositional Logic

Propositional Logic is part of Mathematical Logic. Versions include:

- *First Order Logic* (FOL, a.k.a. *First Order Predicate Calculus* (FOPC)) includes simple term variables and quantifications.
- *Second Order Logic* allows its variables to represent more complex structures (in particular, predicates).
- *Modal Logic* adds support for modalities; that is, concepts such as possibility and necessity.



# Well-Formed Formulae

**Definition: Well-Formed Formula (wff)**

**Example(s):**

# Why Are We Studying Logic?

A few of the many reasons:

- Logic is the foundation for computer operation.
- Logical conditions are common in programs:
  - Selection:

```
if (score <= max) { ... }
```

  - Iteration:

```
while (i<limit && list[i]!=sentinel) ...
```

- All manner of structures in computing have properties that need to be proven (and proofs that need to be understood).
  - Examples: Trees, Graphs, Recursive Algorithms, ...
- Even programs can be proven correct!
- Computational linguistics must represent and reason about human language, and language represents thought (and thus also logic).



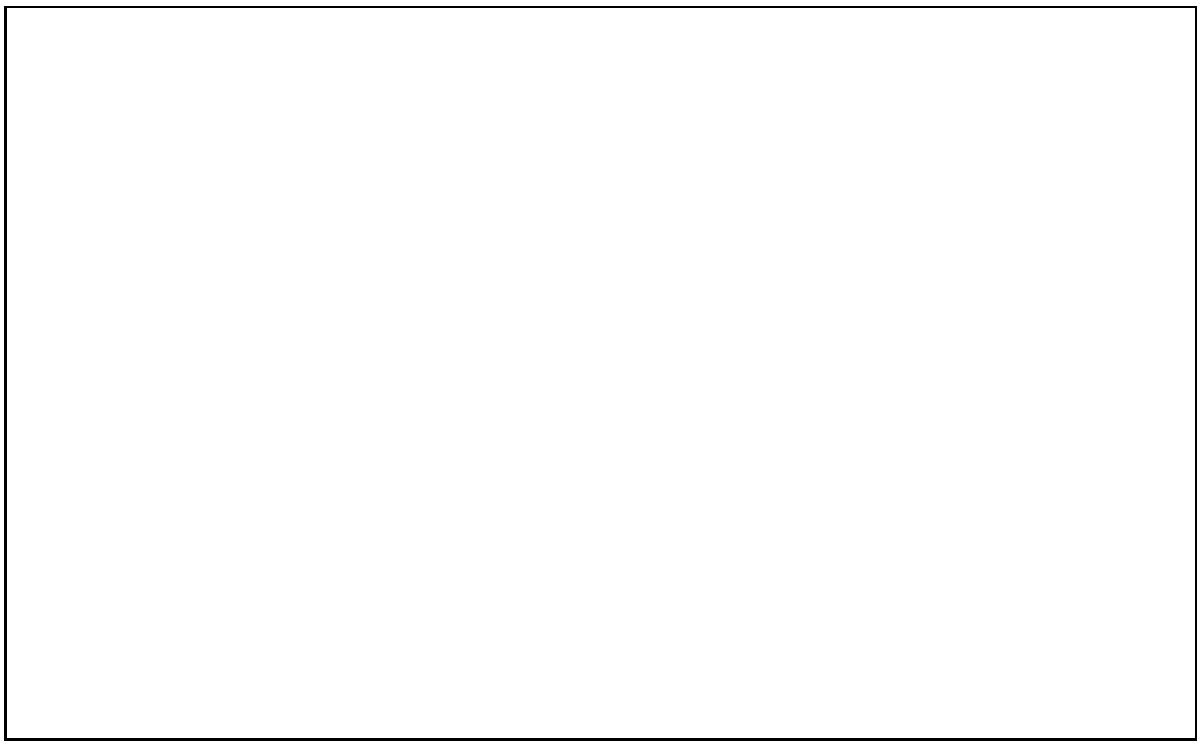
# Simple Propositions (1 / 2)

**Definition: Proposition**

**Definition: Simple Proposition**

# Simple Propositions (2 / 2)

**Example(s):**

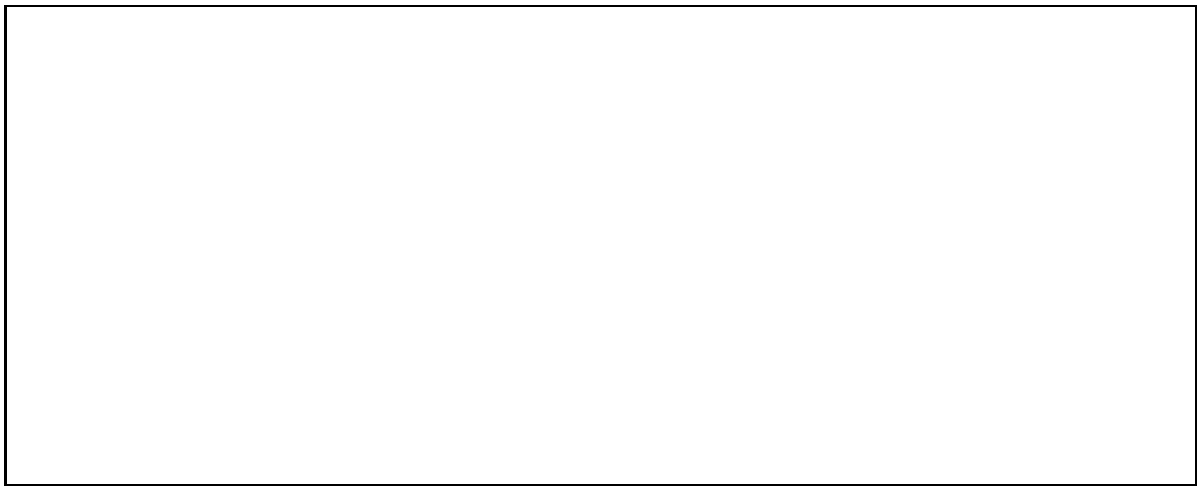




# Proposition Labels

To save writing, it is traditional to label propositions with lower-case letters called *proposition labels* or *statement letters*.

**Example(s):**



# Compound Propositions

**Definition: Compound Proposition**

And with what do we combine them?



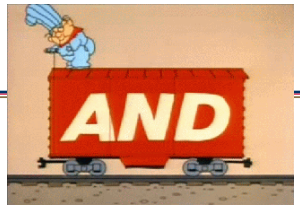
# Conjunctions (1 / 2)

Remember ABC's "Schoolhouse Rock" education series?


"Conjunction Junction" (1973)

(Music/Lyrics by Bob Dorough; Performed by Jack Sheldon)

# Conjunctions (2 / 2)




Conjunctions are:

- compound propositions formed in English with “and” & “but”,
- formed in logic with the caret symbol (“ $\wedge$ ”), and
- true only when both participating propositions are true.

**Example(s):**



# Disjunctions (1 / 3)


Consider this compound proposition:

Under which circumstances is that claim true? Possibilities:

1. The first proposition is true.
2. The second proposition is true.
3. Both of the propositions are true.

If all three are acceptable, the disjunction is

_____ (  ).

# Disjunctions (2 / 3)


Consider the same example and possibilities:

3 is the number of sides of a triangle or the number of times this class meets per week.

Possibilities:

1. The first proposition is true.
2. The second proposition is true.
3. Both of the propositions are true.

If the third possibility is not acceptable, the disjunction is

_____ ( ).



# Disjunctions (3 / 3)

**Example(s):**

# Negation

Negating a proposition simply flips its value.

Common negation notations: $\neg x$ $\overline{x}$ $\sim x$ $x'$

**Example(s):**

**Note:**



# Truth Tables (1 / 2)

Truth tables aid in the evaluation of compound propositions.

Structure of a Truth Table:

| $p$ | $q$ | $p \wedge q$ | $(p \wedge q) \vee p$ |
|---|---|---|---|
| T | T | T | T |
| T | F | F | T |
| F | T | F | F |
| F | F | F | F |

# Truth Tables (2 / 2)

Truth Tables of $\wedge$, $\vee$, $\oplus$, and $\neg$:

NOT ($\neg$)

| $p$ | $\neg p$ |
|---|---|

AND ($\wedge$)

| $p$ | $q$ | $p \wedge q$ |
|---|---|---|

OR ($\vee$)

| $p$ | $q$ | $p \vee q$ |
|---|---|---|

XOR ($\oplus$)

| $p$ | $q$ | $p \oplus q$ |
|---|---|---|



# Precedence of Logical Operators

Total agreement is hard to come by:

| | Rosen 8/e | Gersting 5/e | Hein 2/e | Epp 1/e |
|---|---|---|---|---|
| Precedence | p. 11 | p. 6 | p. 351 | p. 24 |
| Highest | $\neg$ | ’ | $\neg$ | $\sim$ |
| | $\wedge$ | $\wedge$,$\vee$ | $\wedge$ | $\wedge$,$\vee$ |
| $\Updownarrow$ | $\vee$ | $\rightarrow$ | $\vee$ | $\rightarrow$,$\leftrightarrow$ |
| | $\rightarrow$ | $\leftrightarrow$ | $\rightarrow$ | |
| Lowest | $\leftrightarrow$ | | | |

(**Note:** We’ll cover $\rightarrow$ and $\leftrightarrow$ soon.)

In this class:

# Operator Associativity

Consider evaluating: $\mathtt{a} = \mathtt{b} = -2 * 3 * 7;$ in Java

**Example(s):**



# Equivalence of Propositions

**Definition: Logically Equivalent**

**Example(s):**

# Natural Language Stmts $\longrightarrow$ Propositions (1 / 4)

Review: Is There isn't a cloud in the sky a proposition?

**Question: Is the following sentence a proposition?**



# Natural Language Stmts $\longrightarrow$ Propositions (2 / 4)

Step 1: Identify the simple propositions.

> Either Walter deposits his mortgage payment or else he will lose his house and move in with Donna.

Step 2: Assign easy-to-remember statement labels.

# Natural Language Stmts $\longrightarrow$ Propositions (3 / 4)

Step 3: Identify the logical operators.

> Either Walter deposits his mortgage payment or else he will lose his house and move in with Donna.

Step 4: Construct the matching logical expression.



# Natural Language Stmts $\longrightarrow$ Propositions (4 / 4)

So . . . what's the point? Three examples:

- Expressing Program Conditions
- Natural Language Understanding
- Proof Setup

# Three Categories of Propositions (1 / 2)

**Definition: Tautology**

**Definition: Contradiction**

**Definition: Contingency**



# Three Categories of Propositions (2 / 2)

**Example(s):** Which of those is $d \oplus (\neg k \wedge m)$ ?

**Example(s):**

# Aside: Logical Bit Operations in Python/Java

| Operator | Name | Example (Dec.) | Example (Bin.) |
|---|---|---|---|
| $\sim$ | Complement | $\sim 12 = -13$ | $\sim 00001100 = 11110011$ |
| $\&$ | AND | $12 \,\&\, 10 = 8$ | $\begin{array}{r} 1100 \\ \& \; 1010 \\ \hline 1000 \end{array}$ |
| $\mid$ | OR | $12 \mid 10 = 14$ | $\begin{array}{r} 1100 \\ \mid 1010 \\ \hline 1110 \end{array}$ |
| $\wedge$ | XOR | $12 \wedge 10 = 6$ | $\begin{array}{r} 1100 \\ \wedge \; 1010 \\ \hline 0110 \end{array}$ |
| $>>$ | Shift Right | $33 >> 1 = 16$ | $00100001 >> 1 = 00010000$ |
| $<<$ | Shift Left | $33 << 2 = 132$ | $00100001 << 2 = 10000100$ |



# Example: Default Linux File Permissions

```
$ ls -l
-rw-rw-r-- 1 mccann mccann 3561 Oct 28 1929 stocktosell
```

# Conditional Propositions (1 / 3)

**Example:**

**Definition: Conditional Proposition**



# Conditional Propositions (2 / 3)

In "if $p$, then $q$", $p$ and $q$ are known by various names:

Common forms of "if $p$, then $q$" (Rosen 8/e, p. 7):

- if $p$, then $q$
- if $p$, $q$
- $p$ implies $q$
- $p$ only if $q$
- $p$ is sufficient for $q$
- a necessary condition for $p$ is $q$
- $q$ unless $\neg p$
- $q$ if $p$
- $q$ when $p$
- $q$ whenever $p$
- $q$ follows from $p$
- $q$ is necessary for $p$
- a sufficient condition for $q$ is $p$
- $q$ provided that $p$

# Conditional Propositions (3 / 3)

**Example(s):**



# Truth of Conditional Propositions (1 / 2)

When should this be considered 'true'?

If you make it through *voir dire*, you will serve on the jury.

The possibilities:

1. Antecedent true, Consequent true; statement is: ____.
2. Antecedent true, Consequent false; statement is: ____.
3. Antecedent false, Consequent true; statement is: ____.
4. Antecedent false, Consequent false; statement is: ____.

# Truth of Conditional Propositions (2 / 2)

Not satisfied? Maybe this will help:

```
if (y < x) {
    temp = x;  x = y;  y = temp;
}
```



# Inverse, Converse, and Contrapositive

**Definition: Inverse**

**Definition: Converse**

| $r$ | $s$ | |
|---|---|---|
| T | T | |
| T | F | |
| F | T | |
| F | F | |

# Contraposition

**Definition: Contrapositive**

| $r$ | $s$ | |
|---|---|---|
| T | T | |
| T | F | |
| F | T | |
| F | F | |



# Examples: English Translation (1 / 2)

# Examples: English Translation (2 / 2)



# Example: English $\longrightarrow$ Logic

# Another Example: English $\rightarrow$ Logic



# Political Example: "Push" Polling

"What would you think of Elizabeth Colbert Busch if she had done jail time?"

— Asked in telephone calls by Survey Sampling International in the 2013 South Carolina 1st Congressional District special election

# Biconditional Propositions and *iff* (1 / 2)

What is the meaning of:

A triangle is equilateral if and only if all three angles are equal.



# Biconditional Propositions and *iff* (2 / 2)

**Definition: Biconditional Proposition**

| $r$ | $s$ | |
|---|---|---|
| T | T | |
| T | F | |
| F | T | |
| F | F | |

# Biconditionals and Logical Equivalence

## Definition: Logically Equivalent (2)

## Example(s):



# De Morgan's Laws

## Example(s):

# Example: De Morgan's Laws and Programming

Checking to see if a 0–100 numeric score is not a 'B':



# Common Logical Equivalences (1 / 3)

Table I: Some Equivalences using AND ($\wedge$) and OR ($\vee$):

| | | |
|---|---|---|
| (a) | $p \wedge p \equiv p\,, \quad p \vee p \equiv p$ | Idempotent Laws |
| (b) | $p \vee \mathbf{T} \equiv \mathbf{T}\,, \quad p \wedge \mathbf{F} \equiv \mathbf{F}$ | Domination Laws |
| (c) | $p \wedge \mathbf{T} \equiv p\,, \quad p \vee \mathbf{F} \equiv p$ | Identity Laws |
| (d) | $p \wedge q \equiv q \wedge p$<br>$p \vee q \equiv q \vee p$ | Commutative Laws |
| (e) | $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$<br>$(p \vee q) \vee r \equiv p \vee (q \vee r)$ | Associative Laws |
| (f) | $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$<br>$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ | Distributive Laws |
| (g) | $p \wedge (p \vee q) \equiv p$<br>$p \vee (p \wedge q) \equiv p$ | Absorption Laws |

Table II: Some More Equivalences (adding $\neg$):

| | | |
|---|---|---|
| (a) | $\neg(\neg p) \equiv p$ | Double Negation |
| (b) | $p \vee \neg p \equiv \mathbf{T}\,, \quad p \wedge \neg p \equiv \mathbf{F}$ | Negation Laws |
| (c) | $\neg(p \wedge q) \equiv \neg p \vee \neg q$<br>$\neg(p \vee q) \equiv \neg p \wedge \neg q$ | De Morgan's Laws |

# Common Logical Equivalences (2 / 3)

Table III: Still More Equivalences (adding $\rightarrow$):

| | | |
|---|---|---|
| (a) | $p \rightarrow q \equiv \neg p \vee q$ | Law of Implication |
| (b) | $p \rightarrow q \equiv \neg q \rightarrow \neg p$ | Law of the Contrapositive |
| (c) | $\mathbf{T} \rightarrow p \equiv p$ | "Law of the True Antecedent" |
| (d) | $p \rightarrow \mathbf{F} \equiv \neg p$ | "Law of the False Consequent" |
| (e) | $p \rightarrow p \equiv \mathbf{T}$ | Self-implication (a.k.a. Reflexivity) |
| (f) | $p \rightarrow q \equiv (p \wedge \neg q) \rightarrow \mathbf{F}$ | Reductio Ad Absurdum |
| (g) | $\neg p \rightarrow q \equiv p \vee q$ | |
| (h) | $\neg(p \rightarrow q) \equiv p \wedge \neg q$ | |
| (i) | $\neg(p \rightarrow \neg q) \equiv p \wedge q$ | |
| (j) | $(p \rightarrow q) \vee (q \rightarrow p) \equiv \mathbf{T}$ | Totality |
| (k) | $(p \wedge q) \rightarrow r \equiv p \rightarrow (q \rightarrow r)$ | Exportation Law (a.k.a. Currying) |
| (l) | $(p \wedge q) \rightarrow r \equiv (p \rightarrow r) \vee (q \rightarrow r)$ | |
| (m) | $(p \vee q) \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$ | |
| (n) | $p \rightarrow (q \wedge r) \equiv (p \rightarrow q) \wedge (p \rightarrow r)$ | |
| (o) | $p \rightarrow (q \vee r) \equiv (p \rightarrow q) \vee (p \rightarrow r)$ | |
| (p) | $p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \rightarrow r)$ | Commutativity of Antecedents |



# Common Logical Equivalences (3 / 3)

Table IV: Yet More Equivalences (adding $\oplus$ and $\leftrightarrow$):

| | | |
|---|---|---|
| (a) | $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$ | Definition of Biimplication |
| (b) | $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$ | |
| (c) | $p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$ | |
| (d) | $p \oplus q \equiv (p \wedge \neg q) \vee (\neg p \wedge q)$ | Definition of Exclusive Or |
| (e) | $p \oplus q \equiv \neg(p \leftrightarrow q)$ | |
| (f) | $p \oplus q \equiv p \leftrightarrow \neg q \equiv \neg p \leftrightarrow q$ | |

Remember: You **do not** need to memorize these tables . . .

. . . But you **do** need to know how to use them!

# Applications of Logical Equivalences (1 / 5)

**Question: Is $(p \wedge q) \rightarrow p$ a tautology? (1)**

By use of a Truth Table; we've seen this before:

| $p$ | $q$ | $p \wedge q$ | $p$ | $(p \wedge q) \rightarrow p$ |
|---|---|---|---|---|
| T | T | T | T | **T** |
| T | F | F | T | **T** |
| F | T | F | F | **T** |
| F | F | F | F | **T** |

Because the expression evaluates to true for all possible arrangements of truth values, the expression is a tautology.



# Applications of Logical Equivalences (2 / 5)

**Question: Is $(p \wedge q) \rightarrow p$ a tautology? (2)**

# Applications of Logical Equivalences (3 / 5)

**Question: Is $(p \wedge q) \rightarrow p$ a tautology? (3)**



# Applications of Logical Equivalences (4 / 5)

**Example(s):**

```
if ((games <= 10 || ties > 2) && games >= 11) ...
```

# Applications of Logical Equivalences (5 / 5)

**Question: Are** $(p \wedge q) \vee (p \wedge r)$ **and** $p \wedge \overline{(\overline{q} \wedge \overline{r})}$

**logically equivalent?**