

Topic 5:

Direct Proofs of $p \rightarrow q$

Handful O' Definitions (1 / 2)

Definition: Conjecture

Definition: Theorem

Definition: Proof

Handful O' Definitions (2 / 2)

Definition: Lemma

Definition: Corollary

Example(s):

Why do People Fear Proofs?

1. Proofs don't come from an assembly line.
 - ▶ Need knowledge, persistence, and creativity

2. Creating proofs seems like magic.
 - ▶ But they are systematic in many ways

3. Proofs are hard to read and understand.
 - ▶ Only if the writer makes them so

4. Institutionalized Fear.
 - ▶ Many teachers avoid them in classes

Constructing a proof? Remember:

1. There are several proof techniques for a reason.
 - ▶ One may be easier to use than the others
2. Knowledge of mathematics is important.
 - ▶ Remember our Math Review?
3. There are “tricks” to know.
 - ▶ Ex: Dividing an even # in half leaves no remainder
4. Practice helps . . . a lot!
 - ▶ Just as it does for most everything else
5. Dead ends are expected.
 - ▶ Proofs in books are the final, polished versions

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Types Of Proof In This Class

1. Direct Proof
 - ▶ The most common variety
2. Proof by Contraposition
 - ▶ Like Direct, but with a twist
3. Proof by Contradiction
 - ▶ A dark road on a foggy night

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Our First Conjecture

Conjecture: If n is even, then n^2 is also even, $n \in \mathbb{Z}$.

Proof-Writing Miscellanea

- Remember: A conjecture isn't a theorem until proven.
- Don't lose sight of your destination.
- When writing proofs in this class:
 1. Always start with "Proof (*style*):"
 2. Stating your allowed assumptions can help.
 3. Define all introduced variables.
 4. End proofs with "Therefore, " and the conjecture.

[Outside of this class: "Q.E.D." (*quod erat demonstrandum*, Latin for "this was to be demonstrated.")]

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A Conjecture About Inequalities

Conjecture: If $0 < a < b$, then $a^2 < b^2$, $a, b \in \mathbb{R}$.

“Proof By Cases”

Question: How would you prove that $\forall x C(x)$ is true, where $x \in \{6, 28, 496\}$?

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A Direct Proof Employing Cases

Conjecture: $s \rightarrow r \equiv \neg r \rightarrow \neg s$

Proof (direct): Consider all possible combinations of values of r and s :

	r	s	$s \rightarrow r$	$\neg r \rightarrow \neg s$
Case 1:	T	T	T	T
Case 2:	T	F	T	T
Case 3:	F	T	F	F
Case 4:	F	F	T	T

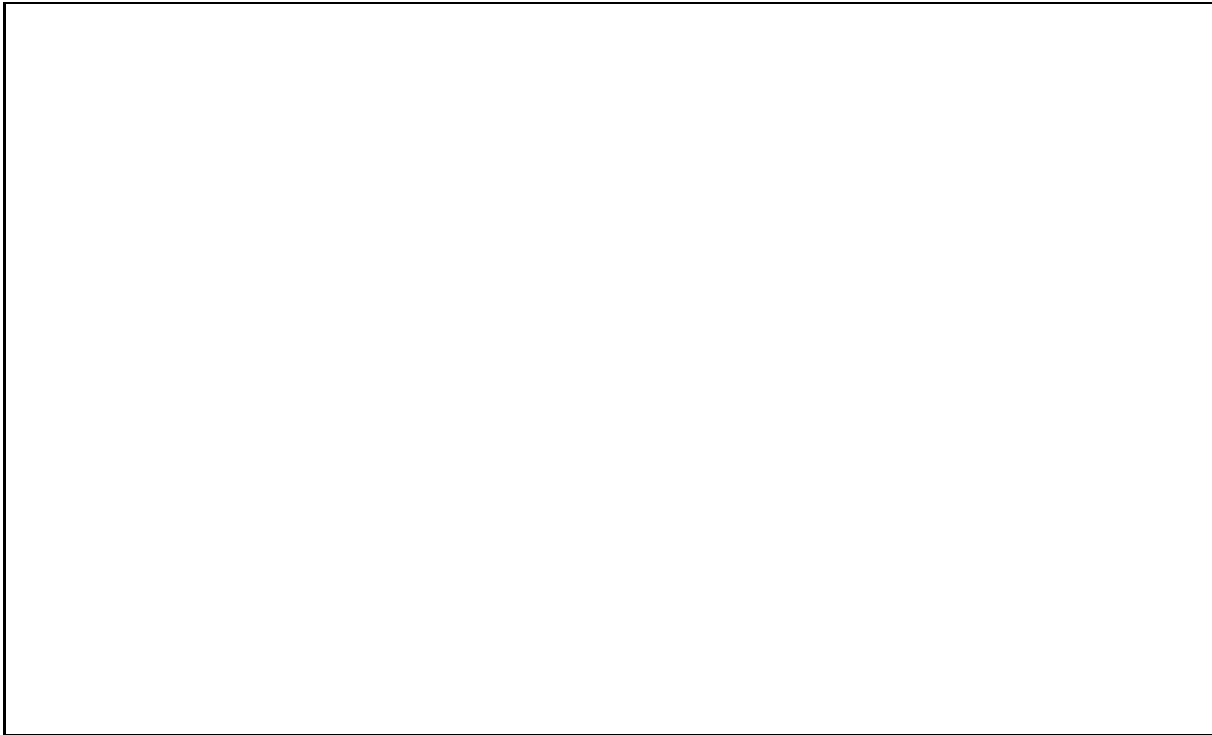
Therefore, $s \rightarrow r \equiv \neg r \rightarrow \neg s$.

(Yes, this truth table is a direct proof by cases.)

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A More Interesting Direct Proof With Cases

Conjecture: $x^2 \% 4 \in \{0, 1\}, x \in \mathbb{Z}$.



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Poor Arguments \longrightarrow Poor Proofs (1 / 2)

Conjecture: $1 < 0$.

Proof or Goof?:

Consider x such that $0 < x < 1$. Take the base-10 logarithm of both sides of $x < 1$: $\log_{10}x < \log_{10}1$. By definition, $\log_{10}1 = 0$. Divide both sides by $\log_{10}x$:

$$\frac{\log_{10}x}{\log_{10}x} < \frac{0}{\log_{10}x}, \text{ which reduces to } 1 < 0.$$

Therefore, $1 < 0$.

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Poor Arguments \longrightarrow Poor Proofs (2 / 2)

Conjecture: For all $n \in \mathbb{Z}^{\text{odd}}$, $(n^2 - 1) \% 4 = 0$.

Proof or Goof?:

Let $x = 1$. $1^2 - 1 = 0$. $0 \% 4 = 0$. Let $x = 3$. $3^2 - 1 = 8$.
 $8 \% 4 = 0$. Let $x = 5$. $5^2 - 1 = 24$. $24 \% 4 = 0$. This
shows no sign of failing to give a result of 0.

Therefore, for all $n \in \mathbb{Z}^{\text{odd}}$, $(n^2 - 1) \% 4 = 0$.

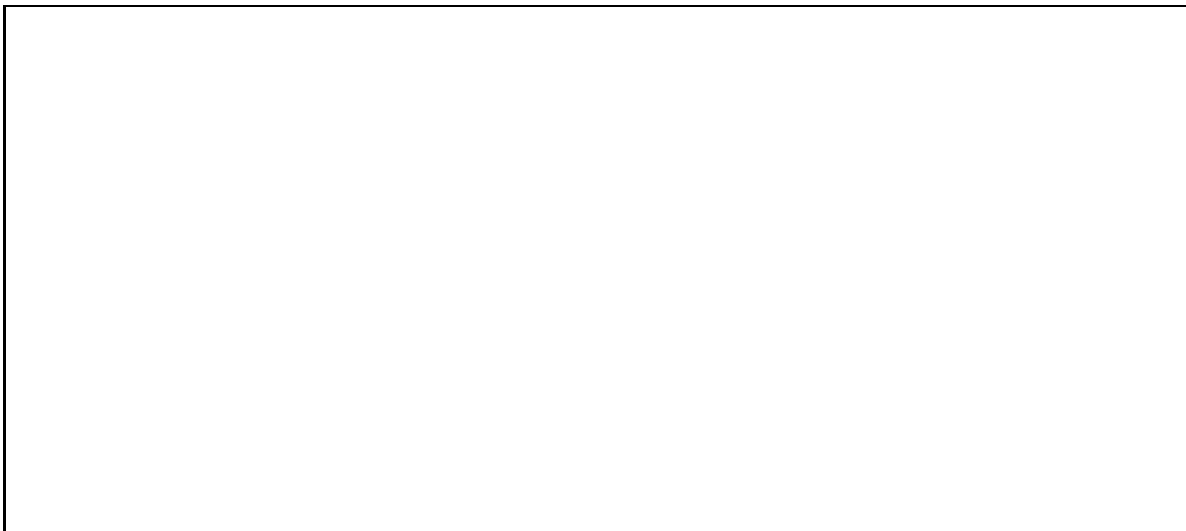
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Disproving Conjectures

Typical approaches:

- (1) Prove that the conjecture's negation is true.
- (2) Find a counter-example. (Very commonly used!)

Example(s):



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