Topic 7:
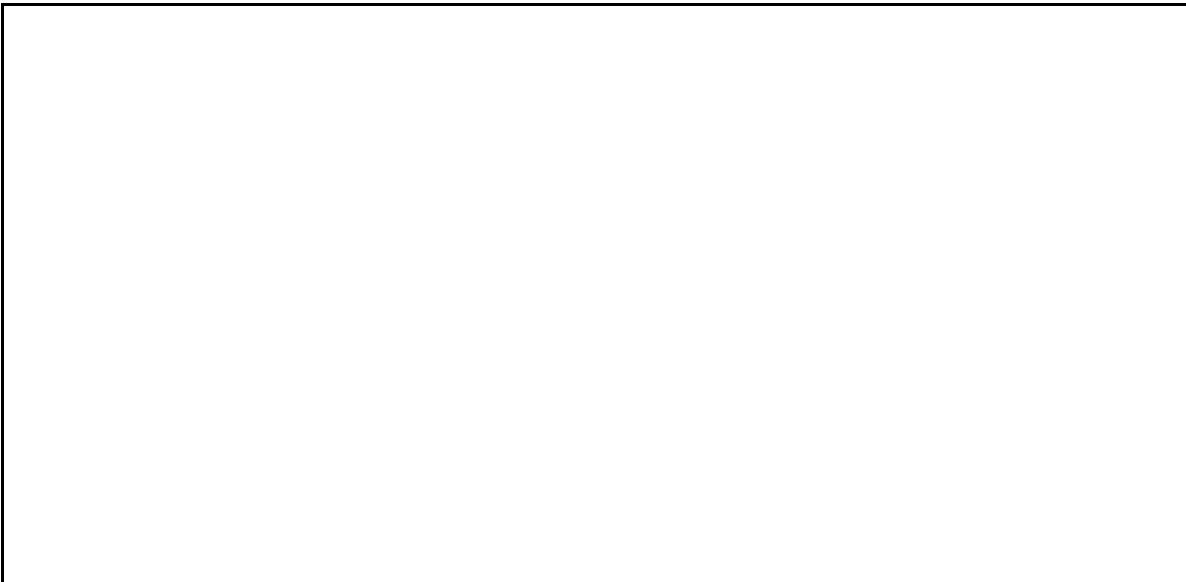
Relations

Background

Having collections of data: Good.

Knowing the connections between collections: Better!

Example(s):

Relations (1 / 2)

Definition: (Binary) Relation

Example(s):

Relations (2 / 2)

Definition: Related

Example(s):

Graph Representations of Relations (1 / 2)

Example #1: Presidents–Parties

Recall: $A = \{$Kennedy, Johnson, Nixon, Carter, Reagan$\}$
$B = \{$Dem, Rep$\}$
$R = \{$(Kennedy, Dem), (Johnson, Dem),
(Nixon,Rep), (Carter, Dem), (Reagan, Rep) $\}$

Kennedy •

Johnson •   • Democratic

Nixon •

Carter •   • Republican

Reagan •

Graph Representations of Relations (2 / 2)

Example #2: $x \mathbin{\%} y = 0, x \neq y$

Recall: $H = \{1, 2, 3, 4, 5, 6\}$
$R = \{(2, 1), (3, 1), (4, 1), (5, 1), (6, 1), (4, 2), (6, 2), (6, 3)\}$

1   2
•   •

6 •   • 3

•   •
5   4

Properties of Relations: Reflexivity

Definition: Reflexivity

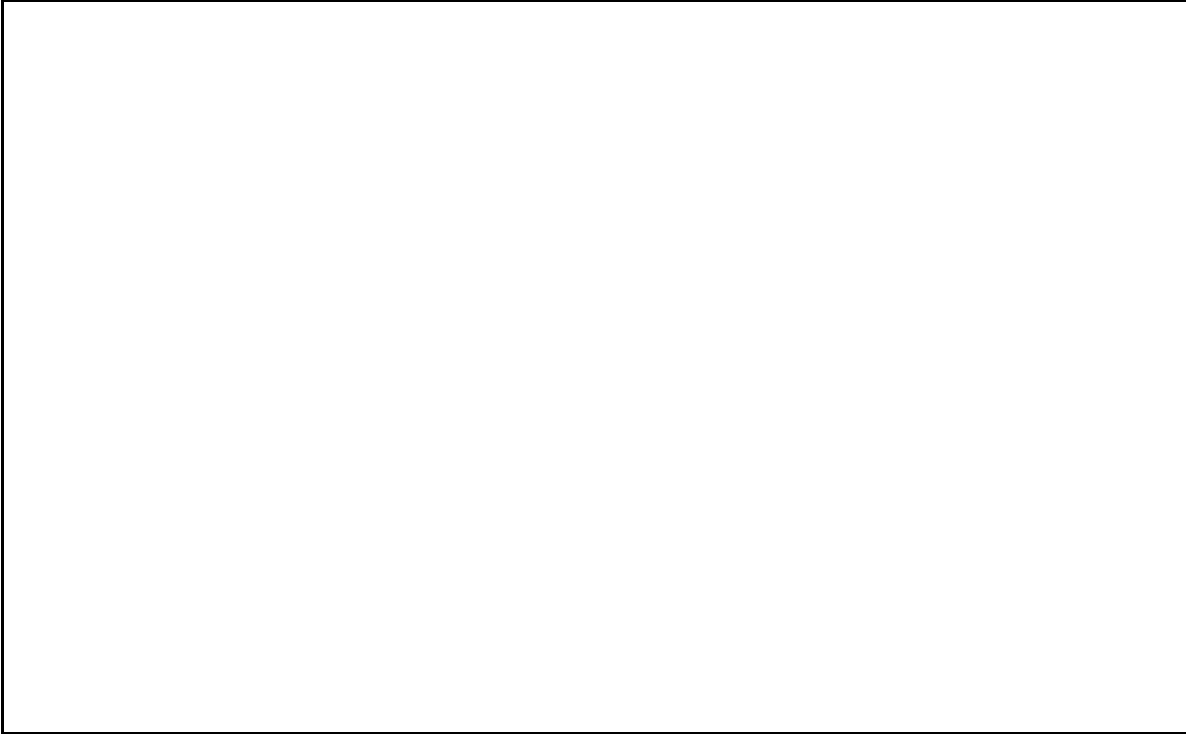**Example(s):**

Properties of Relations: Symmetry (1 / 2)

Definition: Symmetry

Example(s):
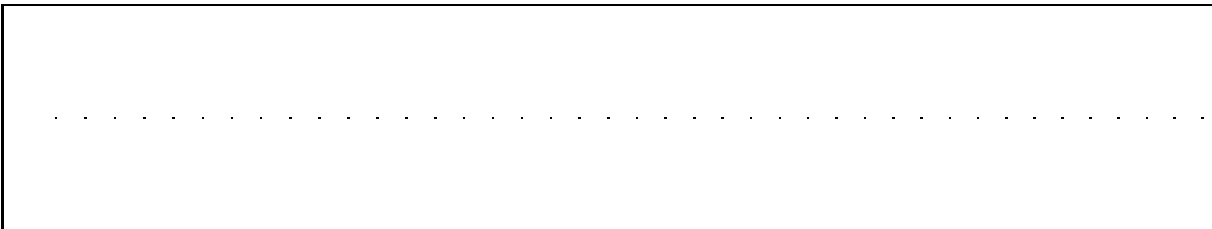
Properties of Relations: Symmetry (2 / 2)

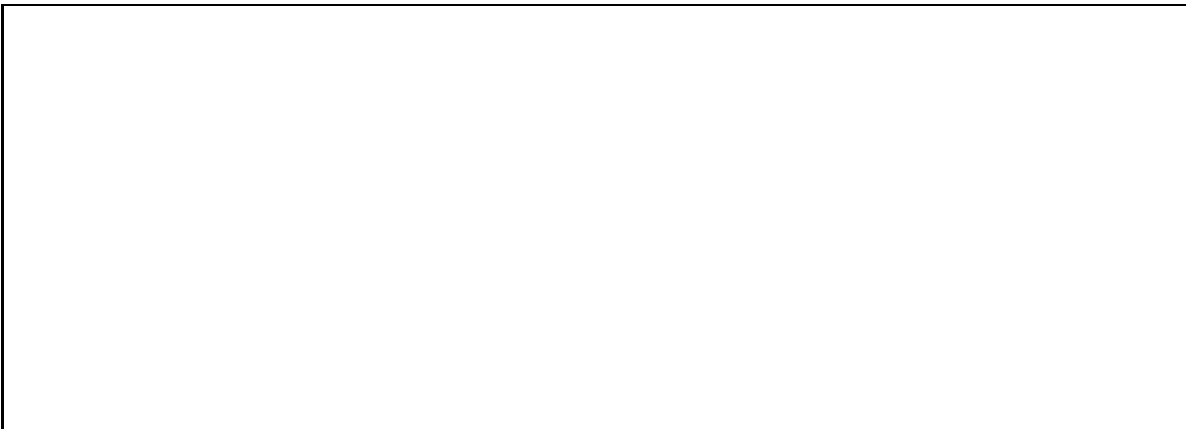**Example(s):** Graph Representations & Symmetry

Properties of Relations: Antisymmetry (1 / 2)

Definition: Antisymmetry

Example(s):

Properties of Relations: Antisymmetry (2 / 2)

Example(s): Graph Representations & Antisymmetry
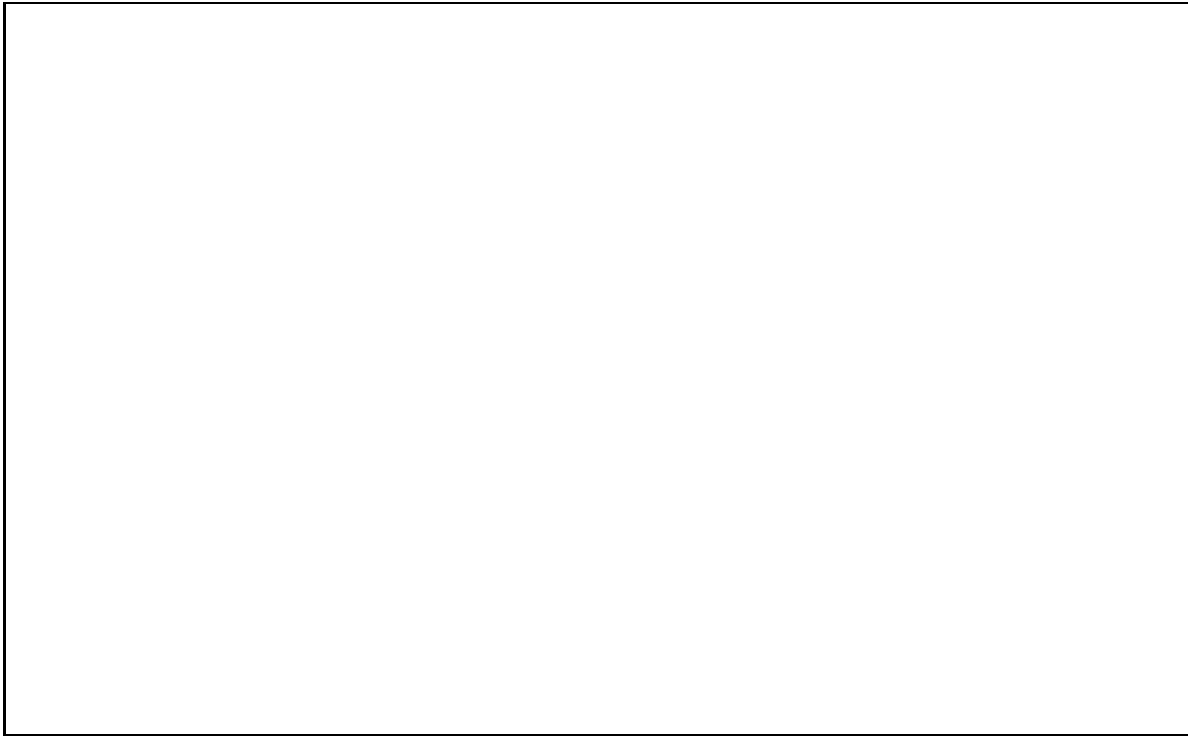
Properties of Relations: Transitivity (1 / 2)

Definition: Transitivity
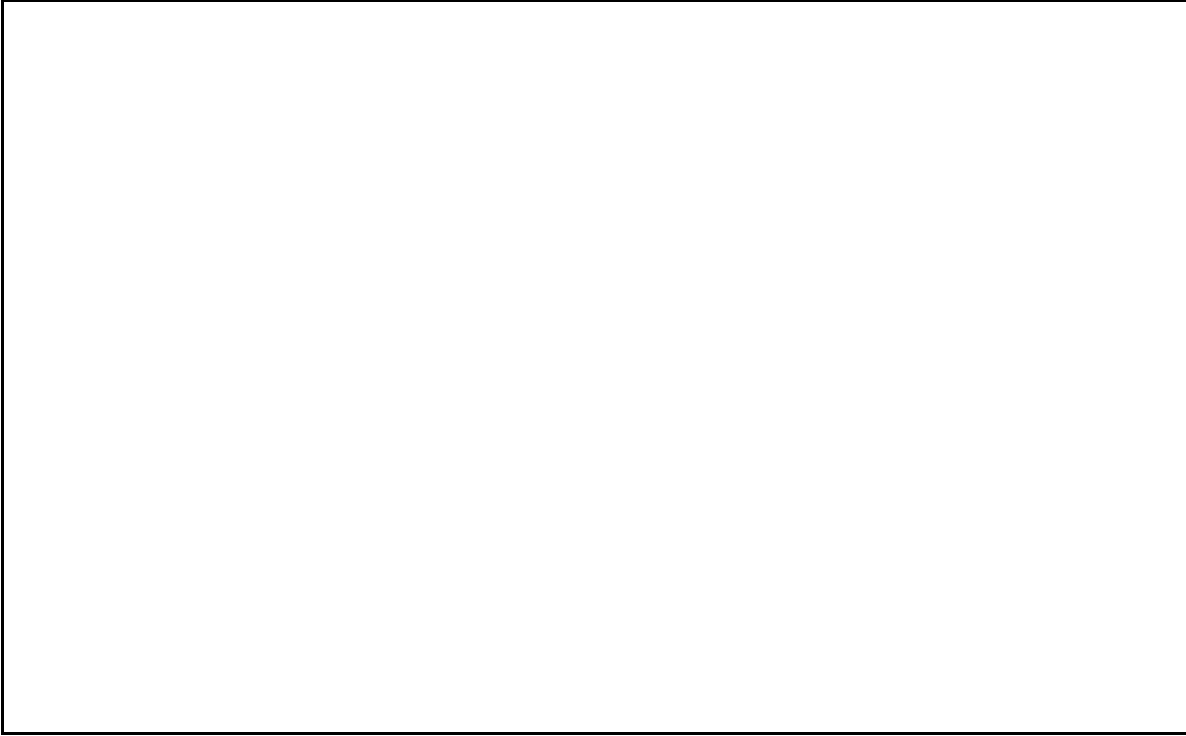
Example(s):

Example(s):

Relational Composition Examples (1 / 4)

Three examples of creating relations from relations.
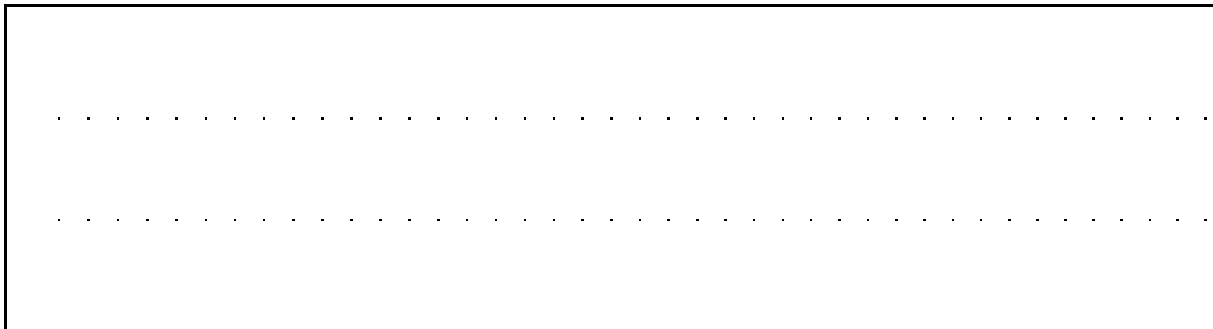
Example #1: Set Operators

Relational Composition Examples (2 / 4)

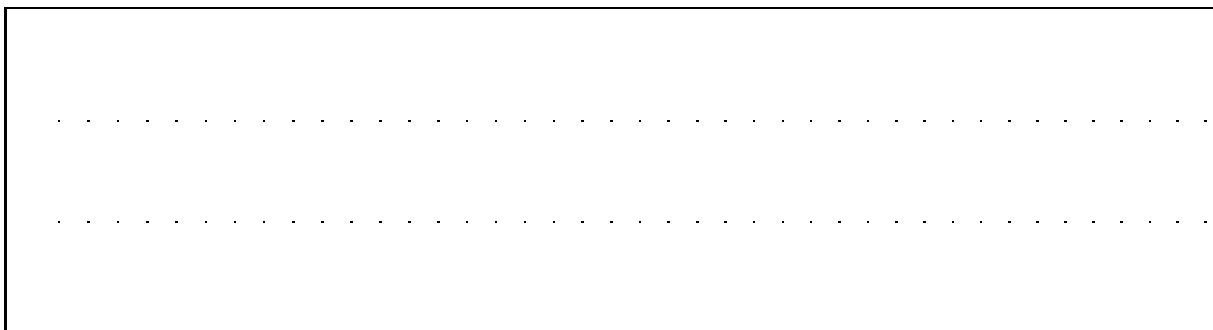Example #2: Swapping content of ordered pairs
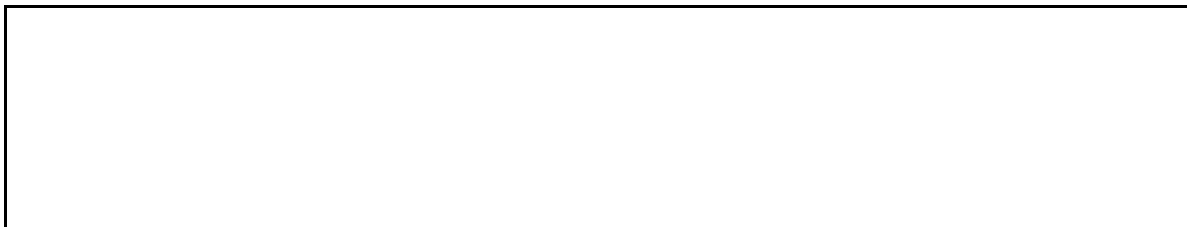
Definition: Inverse

Relational Composition Examples (3 / 4)

Example #3: Composites

Definition: Composite

Example(s):

Relational Composition Examples (4 / 4)

Example #3: Composites (cont.)

Example(s):

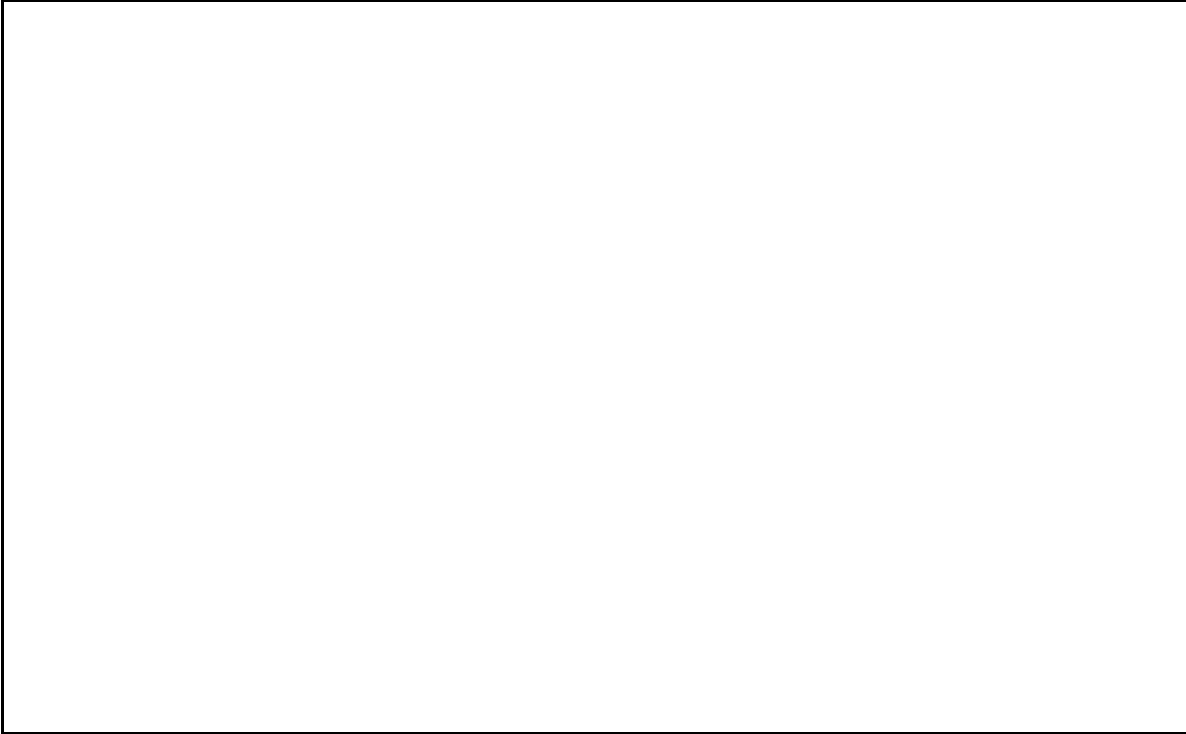**Definition: Complement**

Equivalence Relations (1 / 4)

You may have already implemented one in Java...

Definition: Equivalence Relation

Equivalence Relations (2 / 4)

Example(s):

Equivalence Relations (3 / 4)

So . . . why are these called *equivalence* relations?

Recall:

$$R = \{\ (0,0),$$
$$(1,1), (1,-1), (-1,1), (-1,-1),$$
$$(2,2), (2,-2), (-2,2), (-2,-2)\ \}$$

Equivalence Relations (4 / 4)

Definition: Equivalence Class

. . . .

. . . .

Example(s):

Partial Orders (1 / 3)
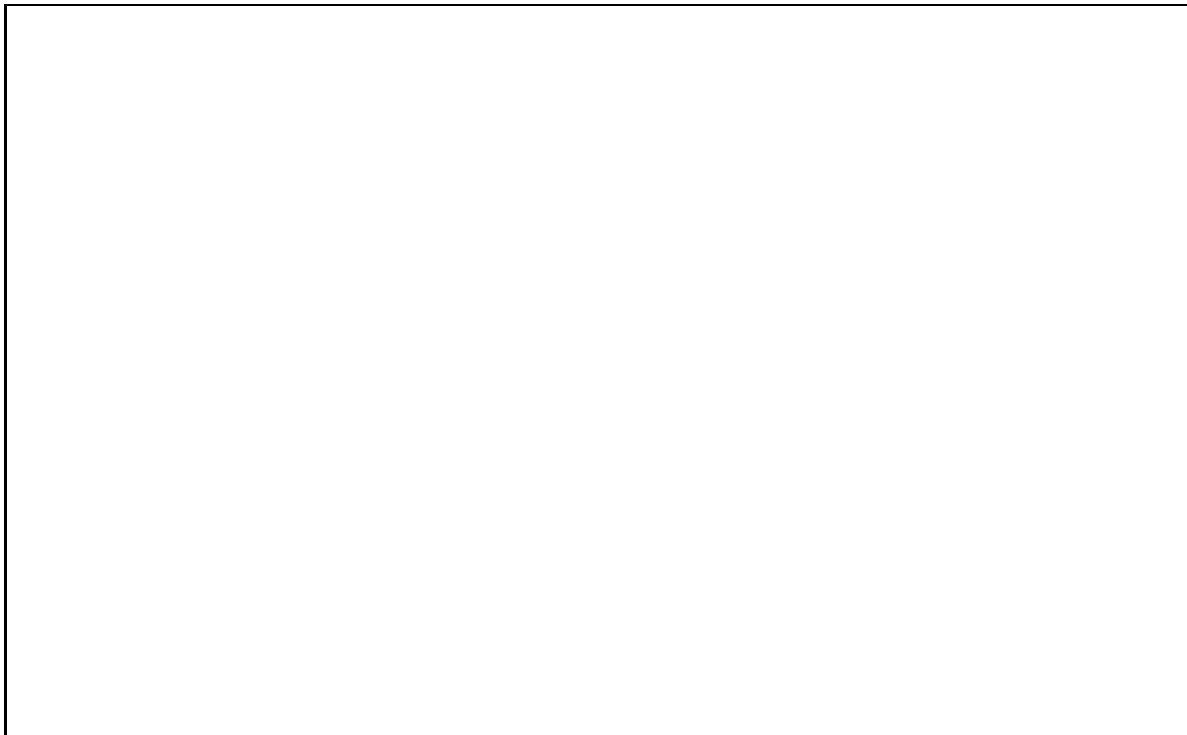
Consider scheduling the construction of a house.

Definition: Reflexive (a.k.a. Weak) Partial Order

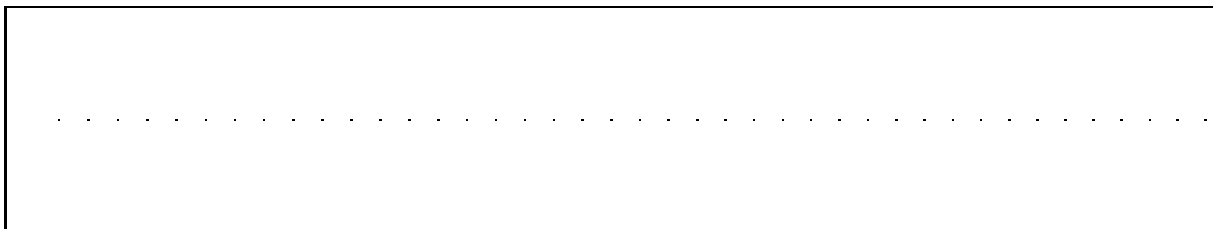. . . . .

Partial Orders (2 / 3)

Example(s):
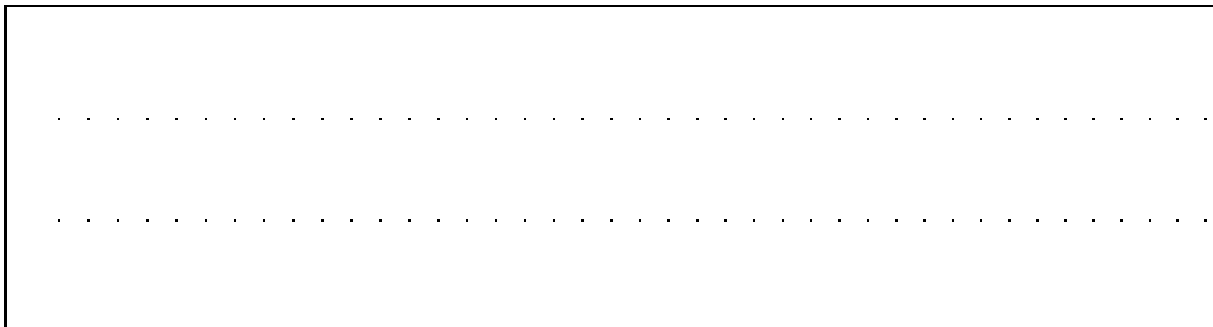
Partial Orders (3 / 3)

Definition: Irreflexivity (of Relations)
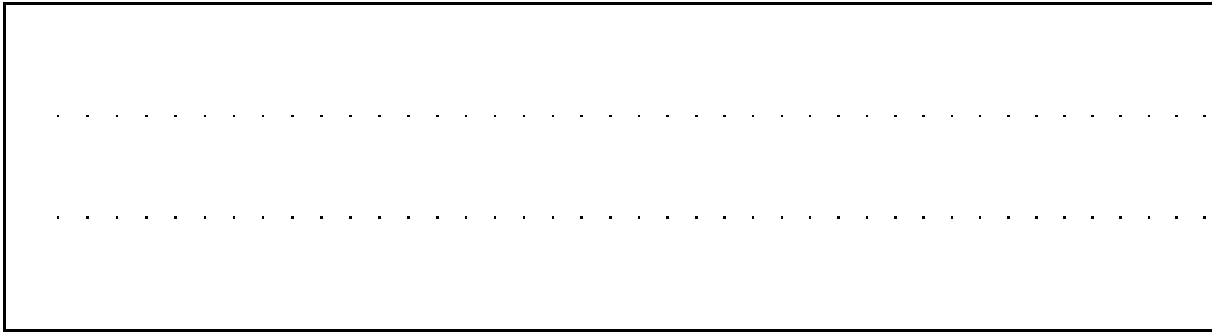
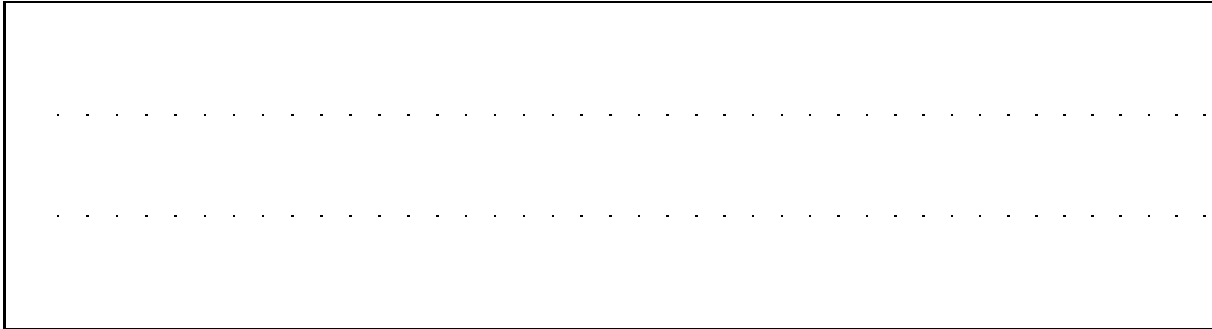**Definition: Irreflexive (a.k.a. Strict) Partial Order**

Total Orders (1 / 2)

Definition: Comparable

Definition: Total Order

Total Orders (2 / 2)

Example(s):