

# Topic 9:

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## Indirect (“Contra”) Proofs of $p \rightarrow q$

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## Review of Direct Proofs

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To prove a conjecture of the form  $p \rightarrow q$  by using a Direct Proof, we:

**Assume** that  $p$  is true, and

**Show** that  $q$ 's truth logically follows.

Reminders:

- If  $p$  is *actually* true, the proof is a sound argument.
- If  $p$  is only *assumed* true, the argument is merely valid.

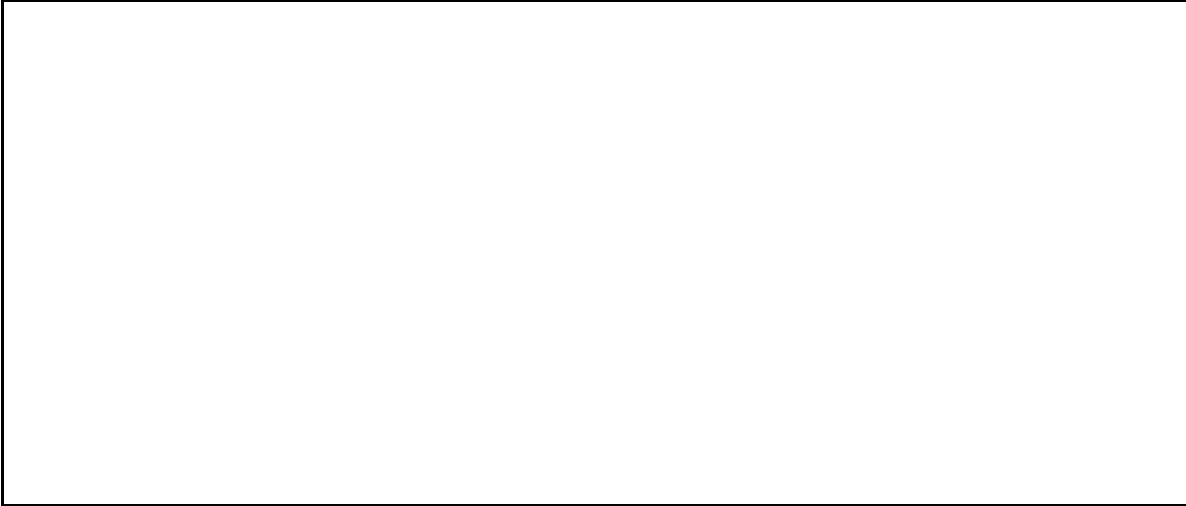
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# “Indirect” Proofs

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We can replace  $p \rightarrow q$  with a logically equivalent form to create additional “indirect” proof techniques.

Example(s):



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## Proof by Contraposition

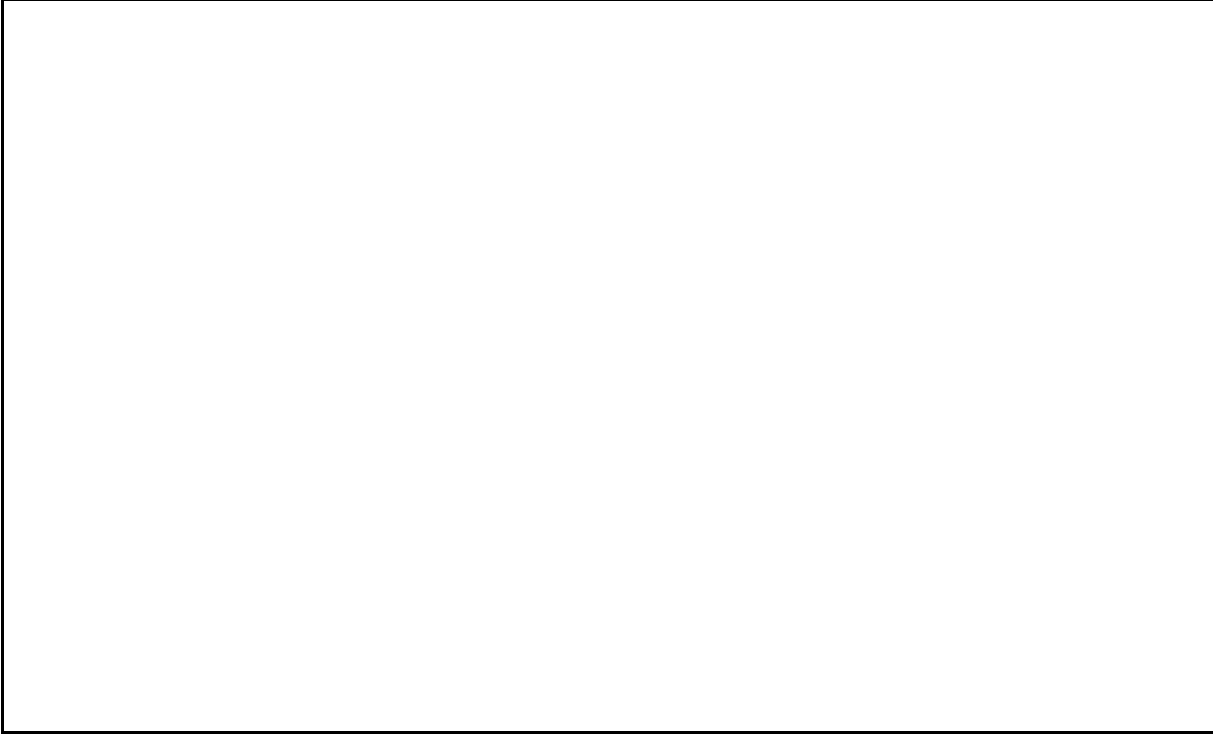
*(a.k.a. Proof of the Contrapositive)*

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## Example #1: Proof by Contraposition

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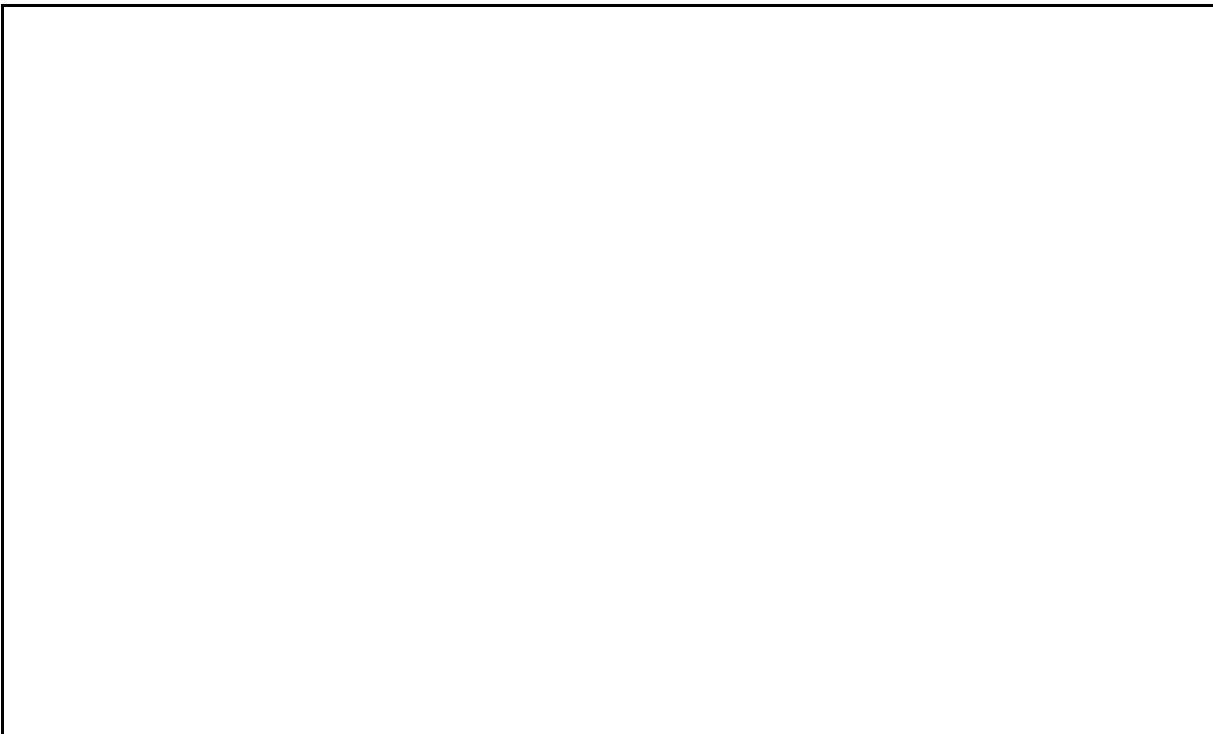
**Conjecture:** If  $ac \leq bc$ , then  $c \leq 0$ , when  $a > b$ .



## Example #2: Proof by Contraposition

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**Conjecture:** If  $n^2$  is even, then  $n$  is even.



# Proof by Contradiction

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(a.k.a. *Reductio ad Absurdum*)

Recall the Law of Implication:  $p \rightarrow q \equiv \neg p \vee q$

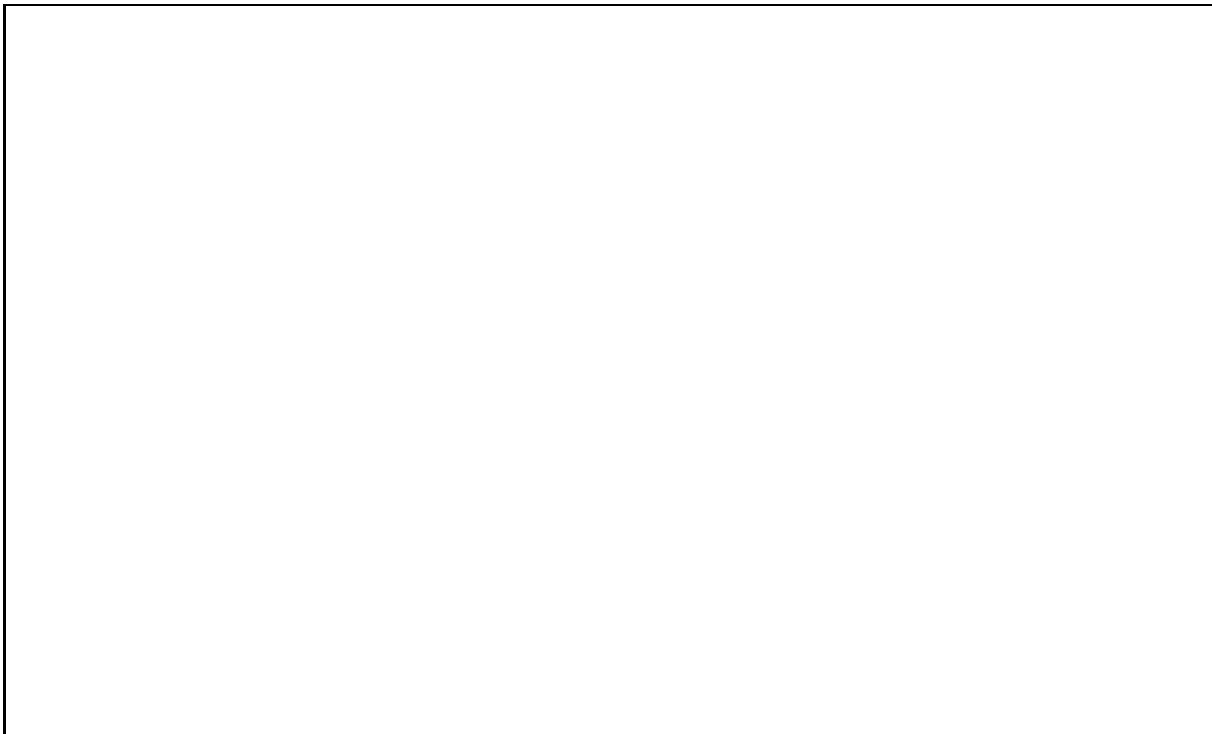
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## Example #1: Proof by Contradiction

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**Conjecture:** If  $3(n - 6)$  is odd, then  $n$  is odd.

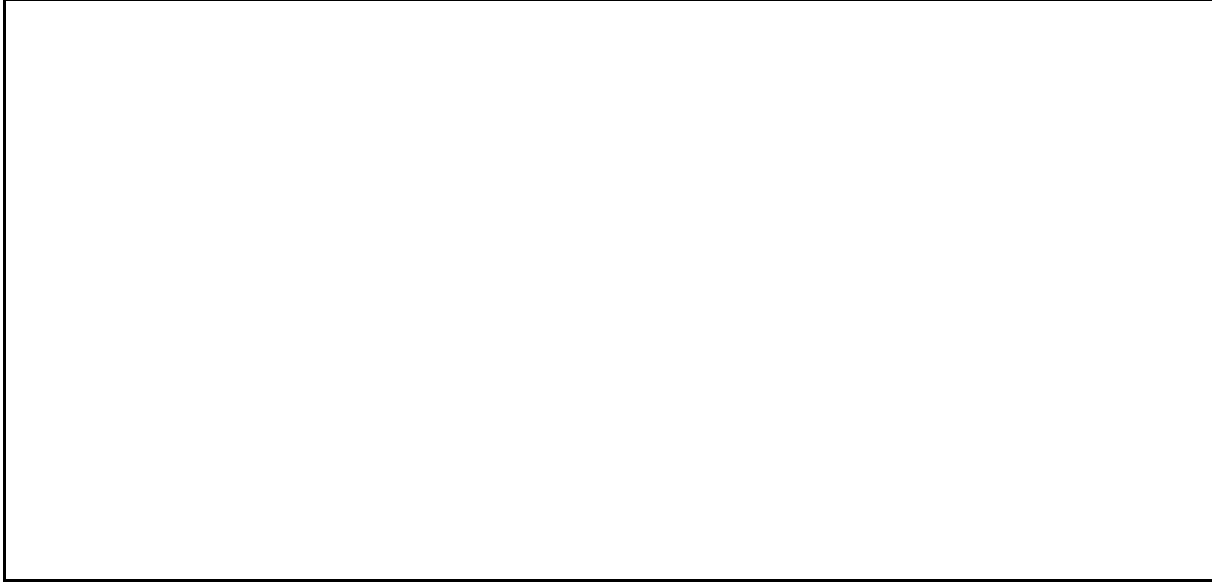


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## Example #2: Proof by Contradiction (1 / 2)

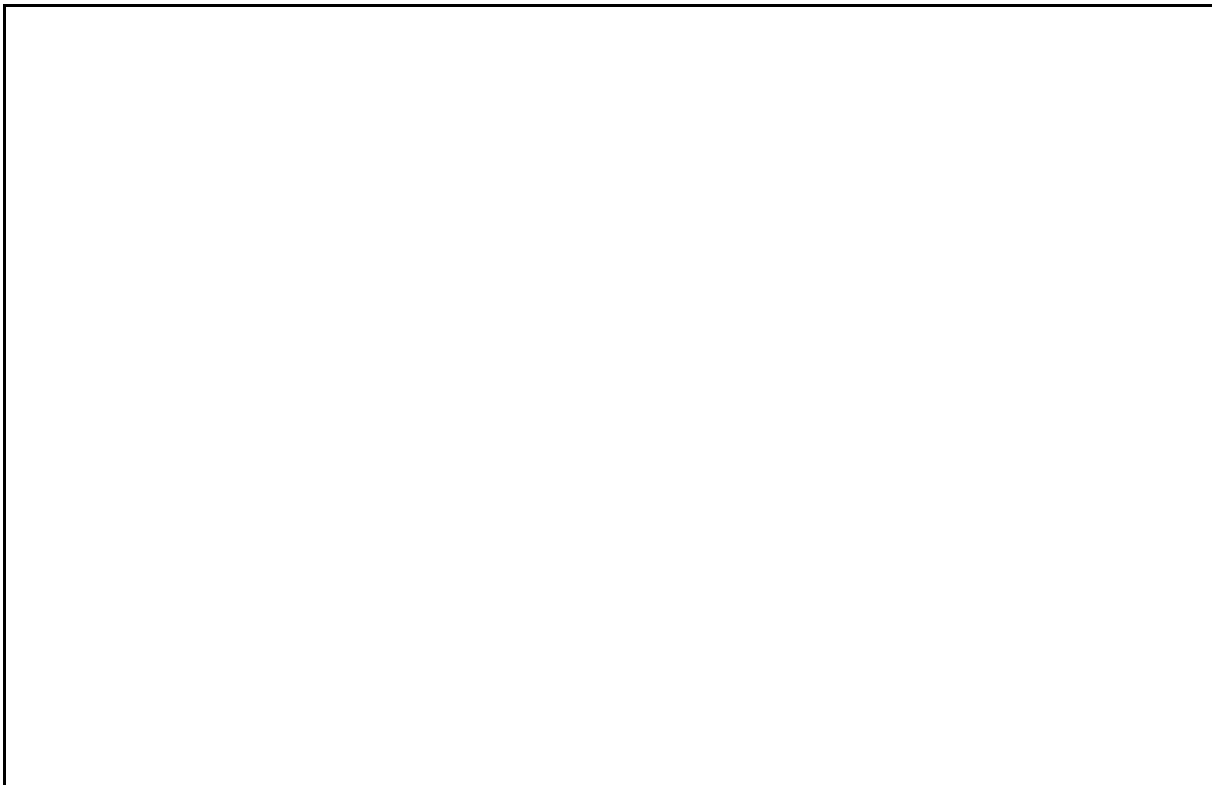
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**Conjecture:** The sum of the squares of two odd integers is never a perfect square. (Or: If  $n = a^2 + b^2$ , then  $n$  is not a perfect square, where  $a, b \in \mathbb{Z}^{odd}$ .)



## Example #2: Proof by Contradiction (2 / 2)

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# How To Prove Biconditional Expressions

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*(i.e., Conjectures Of The Form  $p \leftrightarrow q$ )*

**Example(s):**