Sample First Midterm Exam — The Questions

1. (11 points) Math Review

    (a) What is the symbol for the set of positive integers?

    (b) True or False: 14 is a rational number.

    (c) Evaluate: $\sum_{w=-4}^{2} \frac{w}{2}$

    (d) Create an example that shows subtraction does not distribute over multiplication.

    (e) Simplify: $\log_2((\frac{8}{32})^2)$

2. (21 points) More Math Review!

    (a) What is 443_{10} in both binary and hexadecimal?

    (b) What is $12A_{11}$ in Base 7?

    (c) Simplify to a fraction of the form $\frac{a}{b}$: $1 + \frac{2}{3+\frac{4}{5}}$.

    (d) Let $A = \{b, e, f, g\}$, $B = \{a, c, d, f, g, h\}$, and $C = \{b, d, g\}$, over the universe $\mathcal{U} = \{a, b, c, d, e, f, g, h\}$. What are the results of the evaluations of the expressions $C - (A \cap B)$ and $(\overline{A} \cap C) \cup B$?

    (e) Evalute: $\prod\limits_{a=3}^{3} \sum\limits_{b=10}^{12} (2a + b)$

    (f) Evaluate all three expressions: $7 \mid 15$, $7 \setminus 15$, and $7 \% 15$.

3. (3 points) Define: **Compound Proposition**

4. (6 points) Is $\overline{p \rightarrow q} \equiv p \wedge \overline{q}$? Construct a truth table to determine your answer (and don't forget to tell us what your answer is!).

5. (6 points) <u>Without</u> using a truth table, show that $\overline{p \vee q} \equiv \overline{p \vee (\overline{p} \wedge q)}$ Remember to justify each step.

6. (2 points) What must be done to prove $\forall x P(x), x \in D$ is true, for a given propositional function $P(x)$ and domain D?

7. (12 points) Convert the following English statements into logical notation in terms of the operators, symbols, and quantifiers used in class. Be sure to identify your propositions and predicates; domains are provided.

   (a) Someone is taller than somebody. (domain: people)

   (b) There is no free lunch. (domain: meals)

   (c) "The only consistent people are the dead." (domain: Creatures)

8. (14 points) Convert the following logical expressions to <u>conversational</u> English sentences. Assume that:

$$B(\alpha) : \alpha \text{ is a hummingbird}$$
$$L(\alpha) : \alpha \text{ is large}$$
$$H(\alpha) : \alpha \text{ eats honey}$$
$$C(\alpha) : \alpha \text{ is brightly colored}$$

(a) $\overline{L(Tweety)} \oplus C(Tweety)$

(b) $\exists y(B(y) \wedge \overline{L(y)})$, $y \in$ animals

(c) $\forall z(\overline{H(z)} \rightarrow \overline{C(z)})$, $z \in$ animals

(d) $\forall w(\overline{L(w)} \vee \overline{H(w)})$, $w \in$ birds

9. (8 points) Consider this English sentence, and assume a domain of "Birds" : THERE IS A BIRD THAT IS LARGE, AND IS A HUMMINGBIRD, BUT IS DRAB.

(a) Using the same set of predicates from the previous question, express that sentence as an existentially quantified logical statement.

(b) Express the same sentence in logic, but this time use a quantified logical statement that starts with " $\neg \forall x$ ".

10. (3 points) Fill in the Blanks! We know that there are at least three pairs of terms for p and q in the implication $p \rightarrow q$. Write in the missing name of each pair below.

    (a) p is the _____ and q is the consequent.

    (b) p is the _____ and q is the conclusion.

    (c) p is sufficient and q is _____ .

11. (3 points) Define: **Logical Equivalence** (We covered two definitions in class; either is acceptable.)

12. (6 points) Expression the statement "Exactly two prime numbers are less than five" in logic, using appropriate predicate(s) and domain(s).

Last Revised: January 2023

The Page O' Logical Equivalences ("POLE")

Table I: Some Equivalences using AND ($\land$) and OR ($\lor$):

(a)	$p \land p \equiv p$ $p \lor p \equiv p$	Idempotent Laws
(b)	$p \land \mathbf{F} \equiv \mathbf{F}$ $p \lor \mathbf{T} \equiv \mathbf{T}$	Domination Laws
(c)	$p \land \mathbf{T} \equiv p$ $p \lor \mathbf{F} \equiv p$	Identity Laws
(d)	$p \land q \equiv q \land p$ $p \lor q \equiv q \lor p$	Commutative Laws
(e)	$(p \land q) \land r \equiv p \land (q \land r)$ $(p \lor q) \lor r \equiv p \lor (q \lor r)$	Associative Laws
(f)	$p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$ $p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$	Distributive Laws
(g)	$p \land (p \lor q) \equiv p$ $p \lor (p \land q) \equiv p$	Absorption Laws

Table II: Some More Equivalences (adding Negation ($\neg$)):

(a)	$\neg(\neg p) \equiv p$	Double Negation
(b)	$p \land \neg p \equiv \mathbf{F}$ $p \lor \neg p \equiv \mathbf{T}$	Negation Laws
(c)	$\neg(p \land q) \equiv \neg p \lor \neg q$ $\neg(p \lor q) \equiv \neg p \land \neg q$	De Morgan's Laws

Table III: Still More Equivalences (adding Implication ($\rightarrow$)):

(a)	$p \rightarrow q \equiv \neg p \lor q$	Law of Implication
(b)	$p \rightarrow q \equiv \neg q \rightarrow \neg p$	Law of the Contrapositive
(c)	$\mathbf{T} \rightarrow p \equiv p$	"Law of the True Antecedent"
(d)	$p \rightarrow \mathbf{F} \equiv \neg p$	"Law of the False Consequent"
(e)	$p \rightarrow p \equiv \mathbf{T}$	Self-implication (a.k.a. Reflexivity)
(f)	$p \rightarrow q \equiv (p \land \neg q) \rightarrow \mathbf{F}$	Reductio Ad Absurdum
(g)	$\neg p \rightarrow q \equiv p \lor q$	
(h)	$\neg(p \rightarrow q) \equiv p \land \neg q$	
(i)	$\neg(p \rightarrow \neg q) \equiv p \land q$	
(j)	$(p \rightarrow q) \lor (q \rightarrow p) \equiv \mathbf{T}$	Totality
(k)	$(p \land q) \rightarrow r \equiv p \rightarrow (q \rightarrow r)$	Exportation Law (a.k.a. Currying)
(l)	$(p \land q) \rightarrow r \equiv (p \rightarrow r) \lor (q \rightarrow r)$	
(m)	$(p \lor q) \rightarrow r \equiv (p \rightarrow r) \land (q \rightarrow r)$	
(n)	$p \rightarrow (q \land r) \equiv (p \rightarrow q) \land (p \rightarrow r)$	
(o)	$p \rightarrow (q \lor r) \equiv (p \rightarrow q) \lor (p \rightarrow r)$	
(p)	$p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \rightarrow r)$	Commutativity of Antecedents

Table IV: Yet More Equivalences (adding Exclusive OR ($\oplus$) and Biimplication ($\leftrightarrow$)):

(a)	$p \leftrightarrow q \equiv (p \rightarrow q) \land (q \rightarrow p)$	Definition of Biimplication
(b)	$p \leftrightarrow q \equiv (p \land q) \lor (\neg p \land \neg q)$	
(c)	$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$	
(d)	$p \oplus q \equiv (p \land \neg q) \lor (\neg p \land q)$	Definition of Exclusive Or
(e)	$p \oplus q \equiv \neg(p \leftrightarrow q)$	
(f)	$p \oplus q \equiv p \leftrightarrow \neg q \equiv \neg p \leftrightarrow q$	

Notes:
1. p, q, and r represent arbitrary logical expressions. They may represent equivalent expressions (e.g., if $p \equiv q$, then by Absorption $p \land (p \lor p) \equiv p$).
2. $\mathbf{T}$ and $\mathbf{F}$ represent the logical values True and False, respectively.
3. These tables show many of the common and/or useful logical equivalences; this is not an exhaustive collection!

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