Collected Definitions for Exam #1

I can’t recall the last time I didn’t ask a definition question on a 245 exam. To help you better prepare yourself for such questions, I’ve assembled this list. My pledge to you: If I ask you for a definition on the exam, the term will come from this list. Note that this is not a complete list of the definitions given in class. You should know the others, too, but I won’t specifically ask you for their definitions on the exam.

Once in a while a student will express disappointment that I ask definition questions on exams. My justification is that I think it’s important for you to know what the core terms mean so that you can use them correctly and effectively. At the same time, I don’t require that you memorize the exact wording of the definitions you see here. If you provide a definition in your own words that captures all of the detail found here, that’s fine.

The definitions are grouped by lecture topic, and should be in an order within each topic that is at least close to the order in which the definitions appeared in class.

**Topic 1: Course Background**

- *Discrete Mathematics* is the study of collections of distinct objects.

**Topic 2: Logic**

- *Philosophical Logic* is the classical notion of ‘logic’: The study of thought and reasoning, including arguments and proof techniques.
- *Mathematical Logic* is the use of formal languages and grammars to represent the syntax and semantics of computation.
- A *Well-Formed Formula (wff)* is a correctly structured expression of a language.
- A *proposition* (a.k.a. statement) is a claim that is either true or false with respect to an associated context.
- A *simple proposition* is a proposition containing no logical operators.
- A claim that is a logical combination of multiple simple propositions is a *compound proposition*.
- Two propositions \( p \) and \( q \) are *(Logically) Equivalent* \( (p \equiv q) \) when both evaluate to the same result when presented with the same input. [Note: An alternate, equally-correct definition is given below.]
- A *Tautology* is a proposition that always evaluates to true.
- A *Contradiction* is a proposition that always evaluates to false.
- A *Contingency* is a proposition that is neither a tautology nor a contradiction.
- The *Inverse* of \( p \rightarrow q \) is \( \overline{p} \rightarrow \overline{q} \).
- The *Converse* of \( p \rightarrow q \) is \( q \rightarrow p \).
- The *Contraposition* of \( p \rightarrow q \) is \( \overline{q} \rightarrow \overline{p} \).
- \( p \) and \( q \) are *(Logically) Equivalent*, written \( p \equiv q \), if \( p \leftrightarrow q \) is a tautology. [Note: An alternate, equally-correct definition is given above.]

(Continued . . .)
Topic 3: Quantification

- A statement that includes at least one variable and will evaluate to either true or false when the variable(s) are assigned value(s) is a Predicate (a.k.a. Propositional Function).
- The collection of values from which a variable's value is drawn is known as the Domain of Discourse (a.k.a. Universe of Discourse).
- A quantified variable in a predicate is a Bound variable.
- Unquantified variables are Free (a.k.a. Unbound) variables.
- The Generalized De Morgan’s Laws are the equivalences $\forall x P(x) \equiv \exists x \neg P(x)$ and $\exists x Q(x) \equiv \forall x \neg Q(x)$.

Topic 4: Arguments

- “An Argument is a connected series of statements to establish a definite proposition.” [Credit: Monty Python!]
- An argument that moves from specific observations to a general conclusion is an Inductive Argument.
- An argument that uses accepted general principles to explain a specific situation is a Deductive Argument.
- Any deductive argument of the form $(p_1 \land p_2 \land \ldots \land p_n) \rightarrow q$ is Valid if the conclusion must follow from the hypotheses.
- A valid argument that also has true premises is a Sound argument.
- A Fallacy is an argument constructed with an improper inference.

Topic 5: Proofs of $p \rightarrow q$

- A Conjecture is a statement with an unknown truth value.
- A Theorem is a conjecture that has been shown to be true.
- A sound argument that establishes the truth of a theorem is a Proof.
- A Lemma is a simple theorem whose truth is used to construct more complex theorems.
- A Corollary is a theorem whose truth follows directly from another theorem.