Collected Definitions for Exam #2

I can’t recall the last time I didn’t ask a definition question on an exam. To help you better prepare yourself for such questions, I’ve assembled this list. My pledge to you: If I ask you for a definition on the exam, the term will come from this list. Note that this is not a complete list of the definitions given in class. You should know the others, too, but I won’t specifically ask you for their definitions on the exam.

Once in a while a student will express disappointment that I ask definition questions on exams. My justification is that I think it’s important for you to know what the core terms mean so that you can use them correctly and effectively. At the same time, I don’t require that you memorize the exact wording of the definitions you see here. If you provide a definition in your own words that captures all of the detail found here, without adding anything incorrect, that’s fine.

Topic 5 Carry-Over: “Contra” Proofs and Disproofs

No definitions!  (Yes, there were proof definitions on the first exam, and yes, “contra” proofs (that is, proofs by contraposition and contradiction), biconditional proofs, and disproofs are fair game for this exam, but there are no specific definitions to know for those topics.)

Prolog

• A Horn Clause is a disjunction of predicates in which at most one of the predicates is not negated.

Topic 6: Sets

• A set is an unordered collection of unique objects.
• Set $A$ is a subset of set $B$ (written $A \subseteq B$) if every member of $A$ can be found in $B$.
• $A$ is a proper subset of $B$ (written $A \subset B$) if $A \subseteq B$ and $A \neq B$.
• The power set of a set $A$ (written $\mathcal{P}(A)$) is the set of all of $A$’s subsets, including the empty set.
• Two sets are disjoint if their intersection is $\emptyset$.
• A partition of a set separates all of its members into disjoint subsets.
• An ordered pair is a group of two items $(a, b)$ such that $(a, b) \neq (b, a)$ unless $a = b$.
• The Cartesian Product (symbol: $\times$) of two sets $A$ and $B$ is the set of all ordered pairs $(a, b)$ such that $a \in A$ and $b \in B$.

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**Topic 7: Matrices**

- A **matrix** is an $n$-dimensional collection of values.
- Matrices in which the # of rows equal the # of columns are called **square matrices**.
- Two matrices $A$ and $B$ are **equal** if they share the same dimensions and each pair of corresponding elements is equal.
- The transposition of an $m \times n$ matrix $A$ is an $n \times m$ matrix denoted $A^T$ in which the rows and columns have been exchanged.
- A matrix $A$ is **symmetric** if $A = A^T$.
- The **matrix product** of an $m \times n$ matrix $A$ and an $n \times o$ matrix $B$ is an $m \times o$ matrix $C = A \cdot B$ in which
  \[
  c_{ij} = \sum_{k=1}^{n} (a_{ik} \cdot b_{kj}).
  \]
- The **identity matrix**, denoted $I_n$, is an $n \times n$ matrix populated with ones down the main diagonal (upper-left to lower-right) and zeros elsewhere.
- The $n^{th}$ **power** of a square matrix $A$, denoted $A^n$, is the result of $n-1$ successive matrix products of $A$.
- The $r^{th}$ **Boolean Power** of an $n \times n$ 0-1 matrix $A$, denoted $A^{[r]}$, is the $n \times n$ matrix resulting from $A \odot A \odot \ldots \odot A$, consisting of $r$ $A$'s and $r-1$ boolean products. $A^{[0]} = I_n$.

**Topic 8: Relations**

- A **(binary) relation** from set $X$ to set $Y$ is a subset of the Cartesian Product of the domain $X$ and the codomain $Y$.
- A relation $R$ on set $A$ is **reflexive** if $(a, a) \in R$, $\forall a \in A$.
- A relation $R$ on set $A$ is **symmetric** if $(a, b) \in R$ whenever $(b, a) \in R$ for $a, b \in A$.
- A relation $R$ on set $A$ is **antisymmetric** if $(x, y) \in R$ and $x \neq y$, then $(y, x) \notin R$, $\forall x, y \in A$.
- A relation $R$ on set $A$ is **transitive** if whenever $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$, for $a, b, c \in A$.
- The **inverse** of a relation $R$ on set $A$, denoted $R^{-1}$, contains all of the ordered pairs of $R$ with their components exchanged. (That is, $R^{-1} = \{(b, a) \mid (a, b) \in R\}$.
- Let $G$ be a relation from set $A$ to set $B$, and let $F$ be a relation from $B$ to set $C$. The **composite** of $F$ and $G$, denoted $F \circ G$, is the relation of ordered pairs $(a, c), a \in A, c \in C$, such that $b \in B$, $(a, b) \in G$, and $(b, c) \in F$.
- A relation $R$ on set $A$ is an **equivalence relation** if it is reflexive, symmetric, and transitive.
- The **equivalence class** of an equivalence relation $R$ on set $B$, and an element $b \in B$, is $\{c \mid c \in B \land (b, c) \in R\}$ and is denoted $[b]$.
- A relation $R$ on set $A$ is a **(reflexive/weak) partial order** if it is reflexive, antisymmetric, and transitive.
- A relation $R$ on set $A$ is **irreflexive** if, for all members of $A$, $(a, a) \notin R$.
- A relation $R$ on set $A$ is an **irreflexive (or strict) partial order** if it is irreflexive, antisymmetric, and transitive.
- Let $R$ be a weak partial order on set $A$. $a$ and $b$ are said to be **comparable** if $a, b \in A$ and either $a \leq b$ or $b \leq a$ (that is, $(a, b) \in R$ or $(b, a) \in R$).
- A weak partially-ordered relation $R$ on set $A$ is a **total order** if every pair of elements $a, b \in A$ are comparable.