Collected Definitions Since Exam #3

Here are the definitions that we’ve covered since the material for the last midterm exam. I’m not going to re-print all of the definitions for the whole semester — if you lost a previous exam’s definition handout, you can print another from the class web page.

Topic 13: Counting

- I provided two definitions of the (Generalized) Pigeonhole Principle; learn either one:
  - (a) if $n$ items are placed in $k$ boxes, then at least one box contains at least \( \lceil \frac{n}{k} \rceil \) items.
  - (b) Let $f : X \rightarrow Y$, where $|X| = n$ and $|Y| = k$, and let $m = \lceil \frac{n}{k} \rceil$. There are at least $m$ values $(a_1, a_2, \ldots, a_m)$ such that $f(a_1) = f(a_2) = \ldots = f(a_m)$.
- The Multiplication Principle (a.k.a. the Product Rule): If there are $s$ steps in an activity, with $n_1$ ways of accomplishing the first step, $n_2$ of accomplishing the second, etc., and $n_s$ ways of accomplishing the last step, then there are $n_1 \cdot n_2 \cdot \ldots \cdot n_s$ ways to complete all $s$ steps.
- The Addition Principle (a.k.a. the Sum Rule): If there are $t$ tasks, with $n_1$ ways of accomplishing the first, $n_2$ ways of accomplishing the second, etc., and $n_t$ ways of accomplishing the last, then there are $n_1 + n_2 + \ldots + n_t$ ways to complete one of these tasks, assuming that no two tasks interfere with one another.
- The Principle of Inclusion-Exclusion for Two Sets says that the cardinality of the union of sets $M$ and $N$ is the sum of their individual cardinalities excluding the cardinality of their intersection. That is: $|M \cup N| = |M| + |N| - |M \cap N|$
- The Principle of Inclusion-Exclusion for Three Sets says that the cardinality of the union of sets $M$, $N$, and $O$ is the sum of their individual cardinalities excluding the sum of the cardinalities of their pairwise intersections and including the cardinality of their intersection. That is: $|M \cup N \cup O| = |M| + |N| + |O| - (|M \cap N| + |M \cap O| + |N \cap O|) + |M \cap N \cap O|$
- An ordering of $n$ distinct elements is called a permutation.
- An ordering of an $r$-element subset of $n$ distinct elements is called an $r$-Permutation.
- An $r$-Combination of an $n$-element set $X$ is an $r$-element subset of $X$. The quantity of $r$-element subsets is denoted $C(n, r)$ or \( \binom{n}{r} \), and is read “$n$ choose $r$.”
- A combinatorial proof is an argument based on the principles of counting.

Topic 14: Algorithms

- An algorithm is a finite set of instructions for performing a task.
- A recursive definition has two (sometimes three) parts:
  1. The basis clause determines how trivial cases are to be handled.
  2. The inductive clause explains how complex problems are answered in terms of simpler versions of the same problem.
  3. The extremal clause says that only cases covered by the basis and inductive clauses are covered by the recursive definition. That is, the extremal clause provides boundaries for the definition.
- A recursive algorithm expresses the solution to a task in terms of a simpler case of the same problem.
- The factorial of a non-negative integer $n$, denoted $n!$, is the product of all integer values from 1 through $n$, inclusive. By definition, $0! = 1$.
- The $n^{th}$ term of the Fibonacci sequence is the sum of terms $n - 1$ and $n - 2$, where $F(0) = 0$ and $F(1) = 1$.

(Continued . . .)
Topic 15: Recurrence Relations

• A recurrence relation for the sequence $a_0, a_1, \ldots$ is an equation that expresses term $a_k$ in terms of one or more of its preceding sequence members, one of more of which are explicitly stated initial conditions of the sequence.

• A linear homogeneous recurrence relation with constant coefficients of degree (or order) $k$ (abbreviated: LHRRWCC of degree $k$) has the form $R(n) = c_1 R(n-1) + c_2 R(n-2) + \ldots + c_k R(n-k)$, where $c_i \in \mathbb{R}$ and $c_k \neq 0$.

Topic 16: Finite Probability

• The probability that a specific event will occur is the ratio of the number of occurrences of interest to the number of possible occurrences.