What Is Logic?

**Definition: Philosophical Logic**

**Definition: Mathematical Logic**
Propositional Logic

Propositional Logic is part of Mathematical Logic. Versions include:

- **First Order Logic** (FOL, a.k.a. First Order Predicate Calculus (FOPC)) includes simple term variables and quantifications.

- **Second Order Logic** allows its variables to represent more complex structures (in particular, predicates).

- **Modal Logic** adds support for modalities; that is, concepts such as possibility and necessity.

Well-Formed Formulae

**Definition:** Well-Formed Formula (wff)

Example(s):
Why Are We Studying Logic?

A few of the many reasons:

- Logic is the foundation for computer operation.
- Logical conditions are common in programs:
  - Selection:
    
    ```
    if (score <= max) { ... }
    ```
  - Iteration:
    
    ```
    while (i<limit && list[i]!=sentinel) ...
    ```
- All manner of structures in computing have properties that need to be proven (and proofs that need to be understood).
  - Examples: Trees, Graphs, Recursive Algorithms, ...
- Even programs can be proven correct!
- Computational linguistics must represent and reason about human language, and language represents thought (and thus also logic).

Simple Propositions (1 / 2)

**Definition: Proposition**

- ... ...

**Definition: Simple Proposition**

- ... ...
Proposition Labels

To save writing, it is traditional to label propositions with lower-case letters called *proposition labels* or *statement letters*.

Example(s):
Compound Propositions

**Definition:** Compound Proposition

And with what do we combine them?

Conjunctions (1 / 2)

Remember ABC’s “Schoolhouse Rock” education series?

“Conjunction Junction” (1973)
(Music/Lyrics by Bob Dorough; Performed by Jack Sheldon)
Conjunctions (2 / 2)

Conjunctions are:
- compound propositions formed in English with “and” & “but”,
- formed in logic with the caret symbol (“∧”), and
- true only when both participating propositions are true.

Example(s):

Disjunctions (1 / 3)

Consider this compound proposition:

Under which circumstances is that claim true? Possibilities:

1. The first proposition is true.
2. The second proposition is true.
3. Both of the propositions are true.

If all three are acceptable, the disjunction is

_________________ ( )

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Consider the same example and possibilities:

3 is the number of sides of a triangle or the number of times this class meets per week.

Possibilities:

1. The first proposition is true.
2. The second proposition is true.
3. Both of the propositions are true.

If the third possibility is not acceptable, the disjunction is ______________ ( ).
Negation

Negating a proposition simply flips its value.

Common negation notations: \( \neg x \quad \overline{x} \quad \sim x \quad x' \)

Example(s):

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>p \land q</th>
<th>(p \land q) \lor p</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
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<tr>
<td>T</td>
<td>F</td>
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<td>F</td>
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<td>F</td>
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</tbody>
</table>
Truth Tables (2 / 2)

Truth Tables of $\land$, $\lor$, $\oplus$, and $\neg$:

- **NOT ($\neg$)**
  
<table>
<thead>
<tr>
<th>$p$</th>
<th>$\neg p$</th>
</tr>
</thead>
</table>

- **AND ($\land$)**
  
<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$p \land q$</th>
</tr>
</thead>
</table>

- **OR ($\lor$)**
  
<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$p \lor q$</th>
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</thead>
</table>

- **XOR ($\oplus$)**
  
<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$p \oplus q$</th>
</tr>
</thead>
</table>

Precedence of Logical Operators

Total agreement is hard to come by:

<table>
<thead>
<tr>
<th>Precedence</th>
<th>Rosen 8/e</th>
<th>Gersting 5/e</th>
<th>Hein 2/e</th>
<th>Epp 1/e</th>
</tr>
</thead>
<tbody>
<tr>
<td>Highest</td>
<td>$\neg$</td>
<td>$'$</td>
<td>$\neg$</td>
<td>$\sim$</td>
</tr>
<tr>
<td></td>
<td>$\land$</td>
<td>$\land,\lor$</td>
<td>$\land$</td>
<td>$\land,\lor$</td>
</tr>
<tr>
<td>$\leftrightarrow$</td>
<td>$\lor$</td>
<td>$\rightarrow$</td>
<td>$\lor$</td>
<td>$\rightarrow,\leftrightarrow$</td>
</tr>
<tr>
<td></td>
<td>$\rightarrow$</td>
<td>$\leftrightarrow$</td>
<td>$\rightarrow$</td>
<td></td>
</tr>
<tr>
<td>Lowest</td>
<td>$\leftrightarrow$</td>
<td></td>
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</tbody>
</table>

(Note: We’ll cover $\rightarrow$ and $\leftrightarrow$ soon.)

In this class:
Operator Associativity

Consider evaluating: \[ a = b = -2 \times 3 \times 7; \] in Java

Example(s):
Review: Is There isn’t a cloud in the sky a proposition?

**Question:** Is the following sentence a proposition?

Step 1: Identify the simple propositions.

Either Walter deposits his mortgage payment or else he will lose his house and move in with Donna.

Step 2: Assign easy-to-remember statement labels.
Step 3: Identify the logical operators.

Either Walter deposits his mortgage payment or else he will lose his house and move in with Donna.

Step 4: Construct the matching logical expression.

So . . . what’s the point? Three examples:

- Expressing Program Conditions
- Natural Language Understanding
- Proof Setup
Three Categories of Propositions (1 / 2)

**Definition: Tautology**

**Definition: Contradiction**

**Definition: Contingency**

Example(s): Which of those is \( d \oplus (\neg k \land m) \) ?

Example(s):
### Aside: Logical Bit Operations in Python/Java

<table>
<thead>
<tr>
<th>Operator</th>
<th>Name</th>
<th>Example (Dec.)</th>
<th>Example (Bin.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>~</td>
<td>Complement</td>
<td>~ 12 = −13</td>
<td>~ 00001100 = 11110011</td>
</tr>
<tr>
<td>&amp;</td>
<td>AND</td>
<td>12 &amp; 10 = 8</td>
<td>1100</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>&amp; 1010</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12</td>
<td>10 = 14</td>
</tr>
<tr>
<td></td>
<td>OR</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1110</td>
</tr>
<tr>
<td>^</td>
<td>XOR</td>
<td>12 ^ 10 = 6</td>
<td>1100</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>^ 1010</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0110</td>
</tr>
<tr>
<td>&gt;&gt;</td>
<td>Shift Right</td>
<td>33 &gt;&gt; 1 = 16</td>
<td>00100001 &gt;&gt; 1 = 00010000</td>
</tr>
<tr>
<td>&lt;&lt;</td>
<td>Shift Left</td>
<td>33 &lt;&lt; 2 = 132</td>
<td>00100001 &lt;&lt; 2 = 10000100</td>
</tr>
</tbody>
</table>

### Example: Default Linux File Permissions

```bash
$ ls -l
-rw-rw-r-- 1 mccann mccann 3561 Oct 28 1929 stocktosell
```
Conditional Propositions (1 / 3)

Example:

Definition: Conditional Proposition

Conditional Propositions (2 / 3)

In “if $p$, then $q$”, $p$ and $q$ are known by various names:

Common forms of “if $p$, then $q$” (Rosen 8/e, p. 7):
Truth of Conditional Propositions (1 / 2)

When should this be considered ‘true’?

If you make it through *voir dire*, you will serve on the jury.

The possibilities:

1. Antecedent true, Consequent true; statement is: ____.

2. Antecedent true, Consequent false; statement is: ____.

3. Antecedent false, Consequent true; statement is: ____.

4. Antecedent false, Consequent false; statement is: ____.
Inverse, Converse, and Contrapositive

**Definition: Inverse**

**Definition: Converse**

<table>
<thead>
<tr>
<th>$r$</th>
<th>$s$</th>
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<tbody>
<tr>
<td>T</td>
<td>T</td>
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<tr>
<td>T</td>
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<td>F</td>
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<td>F</td>
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</tbody>
</table>
Contraposition

**Definition: Contrapositive**

<table>
<thead>
<tr>
<th>$r$</th>
<th>$s$</th>
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<tr>
<td>T</td>
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<td>F</td>
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</tbody>
</table>

**Examples: English Translation (1 / 2)**
Example: English → Logic
Political Example: “Push” Polling

“What would you think of Elizabeth Colbert Busch if she had done jail time?”

— Asked in telephone calls by Survey Sampling International in the 2013 South Carolina 1st Congressional District special election
What is the meaning of:

A triangle is equilateral if and only if all three angles are equal.

Definition: Biconditional Proposition

<table>
<thead>
<tr>
<th>T</th>
<th>S</th>
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<tbody>
<tr>
<td>T</td>
<td>T</td>
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<td>F</td>
<td>T</td>
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<tr>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>
Biconditionals and Logical Equivalence

Definition: Logically Equivalent (2)

Example(s):

De Morgan’s Laws

Example(s):
Example: De Morgan’s Laws and Programming

Checking to see if a 0–100 numeric score is not a ‘B’:

---

Common Logical Equivalences (1 / 3)

**Table I: Some Equivalences using AND (\(\land\)) and OR (\(\lor\)):**

| (a) \(p \land p \equiv p\), \(p \lor p \equiv p\) | Idempotent Laws |
| (b) \(p \lor T \equiv T\), \(p \land F \equiv F\) | Domination Laws |
| (c) \(p \land T \equiv p\), \(p \lor F \equiv p\) | Identity Laws |
| (d) \(p \land q \equiv q \land p\), \(p \lor q \equiv q \lor p\) | Commutative Laws |
| (e) \((p \land q) \land r \equiv p \land (q \land r)\), \((p \lor q) \lor r \equiv p \lor (q \lor r)\) | Associative Laws |
| (f) \(p \land (q \lor r) \equiv (p \land q) \lor (p \land r)\), \(p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)\) | Distributive Laws |
| (g) \(p \land (p \lor q) \equiv p\), \(p \lor (p \land q) \equiv p\) | Absorption Laws |

**Table II: Some More Equivalences (adding \(\neg\)):**

| (a) \(\neg(\neg p) \equiv p\) | Double Negation |
| (b) \(p \lor \neg p \equiv T\), \(p \land \neg p \equiv F\) | Negation Laws |
| (c) \(\neg(p \land q) \equiv \neg p \lor \neg q\), \(\neg(p \lor q) \equiv \neg p \land \neg q\) | De Morgan’s Laws |
Common Logical Equivalences (2 / 3)

Table III: Still More Equivalences (adding →):

(a) \( p \rightarrow q \equiv \neg p \lor q \)  
   Law of Implication
(b) \( p \rightarrow q \equiv \neg q \rightarrow \neg p \)  
   Law of the Contrapositive
(c) \( T \rightarrow p \equiv p \)  
   “Law of the True Antecedent”
(d) \( p \rightarrow F \equiv \neg p \)  
   “Law of the False Consequent”
(e) \( p \rightarrow p \equiv T \)  
   Self-implication (a.k.a. Reflexivity)
(f) \( p \rightarrow q \equiv (p \land \neg q) \rightarrow F \)  
   Reductio Ad Absurdum
(g) \( \neg(p \rightarrow q) \equiv p \land \neg q \)  
   Totality
(h) \( (p \land q) \rightarrow r \equiv (p \rightarrow (q \rightarrow r)) \)  
   Exportation Law (a.k.a. Currying)
(i) \( (p \land q) \rightarrow r \equiv (p \rightarrow r) \lor (q \rightarrow r) \)  
   Commutativity of Antecedents
(j) \( (p \lor q) \rightarrow r \equiv (p \rightarrow (q \rightarrow r)) \)  
   Commutativity of Antecedents
(k) \( (p \lor q) \rightarrow r \equiv (p \rightarrow (q \rightarrow r)) \lor (q \rightarrow r) \)  
   Commutativity of Antecedents
(l) \( p \lor q \rightarrow r \equiv (p \rightarrow (q \rightarrow r)) \lor (p \rightarrow r) \)  
   Commutativity of Antecedents
(m) \( p \lor q \rightarrow r \equiv (p \lor q) \land (p \rightarrow r) \)  
   Commutativity of Antecedents
(n) \( p \lor q \rightarrow r \equiv (p \rightarrow (q \rightarrow r)) \lor (q \rightarrow r) \)  
   Commutativity of Antecedents
(o) \( p \lor (q \lor r) \equiv (p \lor q) \lor (q \lor r) \)  
   Commutativity of Antecedents
(p) \( p \lor (q \lor r) \equiv (p \lor q) \lor (p \lor r) \)  
   Commutativity of Antecedents
(q) \( p \lor (q \lor r) \equiv (p \lor q) \lor (q \lor r) \)  
   Commutativity of Antecedents

Remember: You do not need to memorize these tables . . .

. . . But you do need to know how to use them!

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Common Logical Equivalences (3 / 3)

Table IV: Yet More Equivalences (adding \( \oplus \) and \( \leftrightarrow \)):

(a) \( p \leftrightarrow q \equiv (p \rightarrow q) \land (q \rightarrow p) \)  
   Definition of Biimplication
(b) \( p \leftrightarrow q \equiv (p \land q) \lor (\neg p \land \neg q) \)  
   Definition of Exclusive Or
(c) \( p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q \)  
   Definition of Exclusive Or
(d) \( p \oplus q \equiv (p \land \neg q) \lor (\neg p \land q) \)  
   Definition of Exclusive Or
(e) \( p \oplus q \equiv \neg(p \leftrightarrow q) \)  
   Definition of Exclusive Or
(f) \( p \oplus q \equiv p \leftrightarrow \neg q \equiv \neg p \leftrightarrow q \)  
   Definition of Exclusive Or

Remember: You do not need to memorize these tables . . .

. . . But you do need to know how to use them!

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Applications of Logical Equivalences (1 / 5)

**Question:** Is \((p \land q) \rightarrow p\) a tautology? (1)

By use of a Truth Table; we’ve seen this before:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>q</td>
<td>p \land q</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
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<td>F</td>
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<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

Because the expression evaluates to true for all possible arrangements of truth values, the expression is a tautology.

Applications of Logical Equivalences (2 / 5)

**Question:** Is \((p \land q) \rightarrow p\) a tautology? (2)

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Question: Is \((p \land q) \rightarrow p\) a tautology? (3)

Example(s):

```java
if ((games <= 10 || ties > 2) && games >= 11) ...
```
Question: Are \((p \land q) \lor (p \land r)\) and \(p \land (\overline{q} \land \overline{r})\) logically equivalent?