What Is Logic?

**Definition: Philosophical Logic**

... 

**Definition: Mathematical Logic**

...
Propositional Logic

Propositional Logic is part of Mathematical Logic. Versions include:

- **First Order Logic** (FOL, a.k.a. *First Order Predicate Calculus* (FOPC)) includes simple term variables and quantifications.

- **Second Order Logic** allows its variables to represent more complex structures (in particular, predicates).

- **Modal Logic** adds support for modalities; that is, concepts such as possibility and necessity.

---

Well-Formed Formulae

**Definition:** Well-Formed Formula (wff)

\[
\text{...}
\]

**Example(s):**

\[
\text{...}
\]
Why Are We Studying Logic?

A few of the many reasons:

- Logic is the foundation for computer operation.
- Logical conditions are common in programs:
  - Selection:
    ```java
    if (score <= max) { ... }
    ```
  - Iteration:
    ```java
    while (i<limit && list[i]!=sentinel) ...
    ```
- All manner of structures in computing have properties that need to be proven (and proofs that need to be understood).
  - Examples: Trees, Graphs, Recursive Algorithms, ...
- Even programs can be proven correct!
- Computational linguistics must represent and reason about human language, and language represents thought (and thus also logic).

Simple Propositions (1 / 2)

**Definition: Proposition**

```

```

**Definition: Simple Proposition**

```

```
Proposition Labels

To save writing, it is traditional to label propositions with lower-case letters called *proposition labels* or *statement letters*.

Example(s):
Definition: Compound Proposition

And with what do we combine them?

Conjunctions (1 / 2)

Remember ABC’s “Schoolhouse Rock” education series?

“Conjunction Junction” (1973)
(Music/Lyrics by Bob Dorough; Performed by Jack Sheldon)
Conjunctions (2 / 2)

Conjunctions are:

- compound propositions formed in English with “and” & “but”,
- formed in logic with the caret symbol (“∧”), and
- true only when both participating propositions are true.

Example(s):

Disjunctions (1 / 3)

Consider this compound proposition:

Under which circumstances is that claim true? Possibilities:

1. The first proposition is true.
2. The second proposition is true.
3. Both of the propositions are true.

If all three are acceptable, the disjunction is

_______________ ( ).
Disjunctions (2 / 3)

Consider the same example and possibilities:

3 is the number of sides of a triangle or the number of times this class meets per week.

Possibilities:

1. The first proposition is true.
2. The second proposition is true.
3. Both of the propositions are true.

If the third possibility is not acceptable, the disjunction is ______________ ( ).
Negation

Negating a proposition simply flips its value.

Common negation notations: \( \neg x \quad \overline{x} \quad \sim x \quad x' \)

Example(s):

Note:

Truth Tables (1 / 2)

Truth tables aid in the evaluation of compound propositions.

Structure of a Truth Table:

<table>
<thead>
<tr>
<th>( p )</th>
<th>( q )</th>
<th>( p \land q )</th>
<th>( (p \land q) \lor p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>
Truth Tables (2 / 2)

Truth Tables of $\land$, $\lor$, $\oplus$, and $\neg$:

- **NOT ($\neg$)**
  - $p \quad \neg p$

- **AND ($\land$)**
  - $p \quad q \quad p \land q$

- **OR ($\lor$)**
  - $p \quad q \quad p \lor q$

- **XOR ($\oplus$)**
  - $p \quad q \quad p \oplus q$

Precedence of Logical Operators

Total agreement is hard to come by:

<table>
<thead>
<tr>
<th>Precedence</th>
<th>Rosen 8/e</th>
<th>Gersting 5/e</th>
<th>Hein 2/e</th>
<th>Epp 1/e</th>
</tr>
</thead>
<tbody>
<tr>
<td>Highest</td>
<td>$\neg$</td>
<td>'</td>
<td>$\neg$</td>
<td>$\sim$</td>
</tr>
<tr>
<td></td>
<td>$\land$</td>
<td>$\land,\lor$</td>
<td>$\land$</td>
<td>$\land,\lor$</td>
</tr>
<tr>
<td></td>
<td>$\lor$</td>
<td>$\rightarrow$</td>
<td>$\lor$</td>
<td>$\rightarrow,\leftrightarrow$</td>
</tr>
<tr>
<td></td>
<td>$\rightarrow$</td>
<td>$\leftrightarrow$</td>
<td>$\rightarrow$</td>
<td></td>
</tr>
</tbody>
</table>

(Note: We'll cover $\rightarrow$ and $\leftrightarrow$ soon.)

In this class:
Operator Associativity

Consider evaluating: \( a = b = -2 \times 3 \times 7; \) in Java

Example(s):

Equivalence of Propositions

**Definition:** Logically Equivalent

Example(s):
Review: Is **There isn’t a cloud in the sky** a proposition?

**Question:** Is the following sentence a proposition?

```
Either Walter deposits his mortgage payment or else he will lose his house and move in with Donna.
```

**Step 1:** Identify the simple propositions.

**Step 2:** Assign easy-to-remember statement labels.
Step 3: Identify the logical operators.

Either Walter deposits his mortgage payment or else he will lose his house and move in with Donna.

Step 4: Construct the matching logical expression.

Natural Language Stmts → Propositions (4 / 4)

So . . . what’s the point? Three examples:

- Expressing Program Conditions
- Natural Language Understanding
- Proof Setup
Three Categories of Propositions (1 / 2)

Definition: Tautology

Definition: Contradiction

Definition: Contingency

Three Categories of Propositions (2 / 2)

Example(s): Which of those is $d \oplus (\neg k \land m)$?
Aside: Logical Bit Operations in Python/Java

<table>
<thead>
<tr>
<th>Operator</th>
<th>Name</th>
<th>Example (Dec.)</th>
<th>Example (Bin.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>∼</td>
<td>Complement</td>
<td>∼ 12 = −13</td>
<td>∼ 00001100 = 11110011</td>
</tr>
<tr>
<td>&amp;</td>
<td>AND</td>
<td>12 &amp; 10 = 8</td>
<td>1100</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>&amp; 1010</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>OR</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1110</td>
</tr>
<tr>
<td>^</td>
<td>XOR</td>
<td>12 ^ 10 = 6</td>
<td>1100</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>^ 1010</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0110</td>
</tr>
<tr>
<td>&gt;&gt;</td>
<td>Shift Right</td>
<td>33 &gt;&gt; 1 = 16</td>
<td>00100001 &gt;&gt; 1 = 00010000</td>
</tr>
<tr>
<td>&lt;&lt;</td>
<td>Shift Left</td>
<td>33 &lt;&lt; 2 = 132</td>
<td>00100001 &lt;&lt; 2 = 10001000</td>
</tr>
</tbody>
</table>

Example: Default Linux File Permissions

```bash
$ ls -l
-rw-rw-r-- 1 mccann mccann 3561 Oct 28 1929 stocktosell
```
Conditional Propositions (1 / 3)

Example:

Definition: Conditional Proposition

Conditional Propositions (2 / 3)

In “if \( p \), then \( q \)”, \( p \) and \( q \) are known by various names:

Common forms of “if \( p \), then \( q \)” (Rosen 8/e, p. 7):

- if \( p \), then \( q \)
- if \( p \), \( q \)
- \( p \) implies \( q \)
- \( p \) only if \( q \)
- \( p \) is sufficient for \( q \)
- a necessary condition for \( p \) is \( q \)
- \( q \) unless \( \neg p \)
- \( q \) if \( p \)
- \( q \) when \( p \)
- \( q \) whenever \( p \)
- \( q \) follows from \( p \)
- \( q \) is necessary for \( p \)
- a sufficient condition for \( q \) is \( p \)
- \( q \) provided that \( p \)
Truth of Conditional Propositions (1 / 2)

When should this be considered ‘true’?

If you make it through voir dire, you will serve on the jury.

The possibilities:

1. Antecedent true, Consequent true; statement is: ____.

2. Antecedent true, Consequent false; statement is: ____.

3. Antecedent false, Consequent true; statement is: ____.

4. Antecedent false, Consequent false; statement is: ____.
Truth of Conditional Propositions (2 / 2)

Not satisfied? Maybe this will help:

```java
if (y < x) {
    temp = x;  x = y;  y = temp;
}
```

---

Inverse, Converse, and Contrapositive

**Definition: Inverse**

---

**Definition: Converse**

<table>
<thead>
<tr>
<th>( r )</th>
<th>( s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>
Contraposition

Definition: Contrapositive

<table>
<thead>
<tr>
<th>r</th>
<th>s</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

Examples: English Translation (1 / 2)
Example: English → Logic
Another Example: English → Logic

Political Example: “Push” Polling

“What would you think of Elizabeth Colbert Busch if she had done jail time?”

— Asked in telephone calls by Survey Sampling International in the 2013 South Carolina 1st Congressional District special election
What is the meaning of:

A triangle is equilateral if and only if all three angles are equal.

---

**Definition: Biconditional Proposition**

<table>
<thead>
<tr>
<th>T</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>
Biconditionals and Logical Equivalence

Definition: Logically Equivalent (2)

Example(s):

De Morgan’s Laws

Example(s):
Example: De Morgan’s Laws and Programming

Checking to see if a 0–100 numeric score is not a ‘B’:

### Common Logical Equivalences (1 / 3)

**Table I:** Some Equivalences using AND (\(\land\)) and OR (\(\lor\)):

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) &amp;parr;</td>
<td>(p \land p \equiv p), (p \lor p \equiv p)</td>
<td>Idempotent Laws</td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>(p \lor T \equiv T), (p \land F \equiv F)</td>
<td>Domination Laws</td>
<td></td>
</tr>
<tr>
<td>(c)</td>
<td>(p \land T \equiv p), (p \lor F \equiv p)</td>
<td>Identity Laws</td>
<td></td>
</tr>
<tr>
<td>(d)</td>
<td>(p \land q \equiv q \land p), (p \lor q \equiv p \lor q)</td>
<td>Commutative Laws</td>
<td></td>
</tr>
<tr>
<td>(e)</td>
<td>((p \land q) \land r \equiv (p \land q) \land r), ((p \lor q) \lor r \equiv (p \lor q) \lor r)</td>
<td>Associative Laws</td>
<td></td>
</tr>
<tr>
<td>(f)</td>
<td>(p \land (q \lor r) \equiv (p \land q) \lor (p \land r)), (p \lor (q \land r) \equiv (p \lor q) \land (p \lor r))</td>
<td>Distributive Laws</td>
<td></td>
</tr>
<tr>
<td>(g)</td>
<td>(p \land (p \lor q) \equiv p), (p \lor (p \land q) \equiv p)</td>
<td>Absorption Laws</td>
<td></td>
</tr>
</tbody>
</table>

**Table II:** Some More Equivalences (adding \(\neg\)):

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>(\neg (\neg p) \equiv p)</td>
</tr>
<tr>
<td>(b)</td>
<td>(p \lor \neg p \equiv T), (p \land \neg p \equiv F)</td>
</tr>
<tr>
<td>(c)</td>
<td>(\neg (p \land q) \equiv \neg p \lor \neg q), (\neg (p \lor q) \equiv \neg p \land \neg q)</td>
</tr>
</tbody>
</table>
### Table III: Still More Equivalences (adding →):

| (a) | $p \rightarrow q \equiv \neg p \lor q$ | Law of Implication |
| (b) | $p \rightarrow q \equiv \neg q \rightarrow \neg p$ | Law of the Contrapositive |
| (c) | $\top \rightarrow p \equiv p$ | "Law of the True Antecedent" |
| (d) | $p \rightarrow \bot \equiv \neg p$ | "Law of the False Consequent" |
| (e) | $p \rightarrow p \equiv \top$ | Self-implication (a.k.a. Reflexivity) |
| (f) | $p \rightarrow q \equiv (p \land \neg q) \rightarrow \bot$ | Reductio Ad Absurdum |
| (g) | $\neg(p \rightarrow q) \equiv p \land \neg q$ |
| (h) | $\neg(p \rightarrow q) \equiv p \land \neg q$ |
| (i) | $\neg(p \rightarrow \neg q) \equiv p \land q$ |
| (j) | $(p \rightarrow q) \lor (q \rightarrow p) \equiv \top$ | Totality |
| (k) | $(p \land q) \rightarrow r \equiv p \rightarrow (q \rightarrow r)$ |
| (l) | $(p \land q) \rightarrow r \equiv (p \rightarrow r) \lor (q \rightarrow r)$ |
| (m) | $(p \lor q) \rightarrow r \equiv (p \rightarrow r) \land (q \rightarrow r)$ |
| (n) | $p \rightarrow (q \land r) \equiv (p \rightarrow q) \land (p \rightarrow r)$ |
| (o) | $p \rightarrow (q \lor r) \equiv (p \rightarrow q) \lor (p \rightarrow r)$ |
| (p) | $p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \rightarrow r)$ | Commutativity of Antecedents |

### Table IV: Yet More Equivalences (adding ⊕ and ↔):

| (a) | $p \leftrightarrow q \equiv (p \rightarrow q) \land (q \rightarrow p)$ | Definition of Biimplication |
| (b) | $p \leftrightarrow q \equiv (p \land q) \lor (\neg p \land \neg q)$ |
| (c) | $p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$ |
| (d) | $p \oplus q \equiv (p \land \neg q) \lor (\neg p \land q)$ | Definition of Exclusive Or |
| (e) | $p \oplus q \equiv \neg(p \leftrightarrow q)$ |
| (f) | $p \oplus q \equiv p \leftrightarrow \neg q \equiv \neg p \leftrightarrow q$ |

**Remember:** You **do not** need to memorize these tables . . .

. . . But you **do** need to know how to use them!
Question: Is \((p ∧ q) \rightarrow p\) a tautology? (1)

By use of a Truth Table; we’ve seen this before:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>(p \land q)</th>
<th>(p)</th>
<th>((p \land q) \rightarrow p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>

Because the expression evaluates to true for all possible arrangements of truth values, the expression is a tautology.

Applications of Logical Equivalences (2 / 5)

Question: Is \((p ∧ q) \rightarrow p\) a tautology? (2)
Question: Is \((p \land q) \rightarrow p\) a tautology? (3)

Example(s):

```java
if ((games <= 10 || ties > 2) && games >= 11) ...
```
Question: Are \((p \land q) \lor (p \land r)\) and \(p \land (\overline{q} \land \overline{r})\) logically equivalent?