Topic 4:

Arguments

The Logical Kind, Not The Talk Radio Kind.

Monty Python’s “The Argument Clinic”

Featuring:

- Michael Palin as “Man”
- Rita Davies as “Receptionist”
- Graham Chapman as “Mr. Barnard”
- John Cleese as “Mr. Vibrating”
- Eric Idle as “Complainer”
- Terry Jones as “Spreaders”

Definition: **Argument**
Inductive and Deductive Reasoning (1 / 3)

**Definition:** Inductive Argument

**Definition:** Deductive Argument

Inductive and Deductive Reasoning (2 / 3)

**Example(s):**
What type of argument is this?

3 is a prime number, 5 is a prime number, and 7 is a prime number. Therefore, all positive odd integers above 1 are prime numbers.

Structure of a Deductive Argument

\((p_1 \land p_2 \land \ldots \land p_n) \rightarrow q\)
# Valid and Sound Arguments (1 / 2)

**Definition: Valid Argument**

... ...

**Example(s):**

... ...

# Valid and Sound Arguments (2 / 2)

**Example(s):**

... ...

**Definition: Sound Argument**

... ...
Some Rules of Inference (1 / 2)

Learn these!

1. **Addition**

2. **Simplification**

3. **Conjunction**

4. **Modus Ponens**

Some Rules of Inference (2 / 2)

Learn these, too!

5. **Modus Tollens**

6. **Hypothetical Syllogism**

7. **Disjunctive Syllogism**

8. **Resolution**
#1: You accidently drop a pen. You know that the pen will fall if it is dropped. How do you know that the pen will fall?

#2: If $191$ is divisible by $7$, then $191^2$ is divisible by $49$.
$191$ is divisible by $7$, so $191^2$ is divisible by $49$.

Is this argument valid?
Examples of Valid Arguments (3 / 4)

#3: If you email me a love note, I’ll send you flowers. If you don’t, I’ll study Discrete Math. If I study Discrete Math, I’ll do well on the quiz.

Can we conclude that, if I don’t send you flowers, I’ll do well on the quiz?

Examples of Valid Arguments (4 / 4)

#3: (cont.)

\[ p: \text{You email me a love note} \]
\[ q: \text{I send you flowers} \]
\[ r: \text{I study Discrete Math} \]
\[ s: \text{I do well on the quiz} \]

\[ p \rightarrow q \]
\[ \neg p \rightarrow r \]
\[ r \rightarrow s \]

\[ \therefore \quad \neg q \rightarrow s \quad ??? \]
Four common rules:

1. **Universal Instantiation**
   \[ \forall x \ P(x), \ x \in D \ / \ \therefore \ P(d) \text{ if } d \in D \]

2. **Universal Generalization**
   \[ P(d) \text{ for any } d \in D \ / \ \therefore \ \forall x \ P(x), \ x \in D \]

3. **Existential Instantiation**
   \[ \exists x \ P(x), \ x \in D \ / \ \therefore \ P(d) \text{ for some } d \in D \]

4. **Existential Generalization**
   \[ P(d) \text{ for some } d \in D \ / \ \therefore \ \exists x \ P(x), \ x \in D \]
Definition: Fallacy

Three classic types:

1. Affirming the Conclusion (or . . . Consequent)
2. Denying the Hypothesis (or . . . Antecedent)
3. Begging the Question (a.k.a. Circular Reasoning)
Fallacies for Fun

1. Fallacy of Interrogation

2. ‘No True Scotsman’ Fallacy

Extra Slides

The remaining slides in this topic are some that I no longer cover in class. I won’t ask about them on a quiz or an exam, but they could be referenced on a homework or in section.
Specious Reasoning: The Bear Patrol (1 / 3)

Homer: Ah, not a bear in sight. The Bear Patrol must be working like a charm!

Lisa: That's specious reasoning, Dad. [...] By your logic, I could claim that this rock keeps tigers away!

Homer: Oh . . . and how does it work?

Lisa: It doesn't work. [...] It's just a stupid rock. [...] But I don’t see any tigers around here, do you?

Homer: Lisa, I want to buy your rock.

Specious Reasoning: The Bear Patrol (2 / 3)

Definition: Specious Reasoning

An unsupported or improperly constructed argument. (That is, an unsound or invalid argument.)

Question: Where is the error in Homer’s logic?

First issue: Which of these is Homer’s argument?

\[
\begin{align*}
(1) & \quad \neg b \quad \text{(Given)} \\
(2) & \quad \therefore w \quad (???)
\end{align*}
\]

\[
\begin{align*}
(1) & \quad w \quad \text{(Given)} \\
(2) & \quad \therefore \neg b \quad (???)
\end{align*}
\]

The first seems most reasonable in context.
Question: Where is the error in Homer’s logic? (cont.)

Next, what is the missing piece of Homer’s argument?

(1) \(\neg b\)
(2) \(\neg b \rightarrow w\) ← this is what we’re trying to show!

(3) \(\therefore w\) (1, 2, Modus Ponens)

OK, then, how about . . .

(1) \(\neg b\)
(2) \(w \rightarrow \neg b\) ← might sound good, but . . .

(3) \(\therefore w\) (1, 2, um . . . Abracadabra?)

(The second form of Homer’s argument fails similarly.)