Topic 4: Arguments

* The Logical Kind, Not The Talk Radio Kind.

Monty Python’s “The Argument Clinic”

Featuring:

- Michael Palin as “Man”
- Rita Davies as “Receptionist”
- Graham Chapman as “Mr. Barnard”
- John Cleese as “Mr. Vibrating”
- Eric Idle as “Complainer”
- Terry Jones as “Spreaders”

Definition: Argument
Inductive and Deductive Reasoning (1 / 3)

**Definition: Inductive Argument**


**Definition: Deductive Argument**


Inductive and Deductive Reasoning (2 / 3)

**Example(s):**
What type of argument is this?

3 is a prime number, 5 is a prime number, and 7 is a prime number. Therefore, all positive odd integers above 1 are prime numbers.

Structure of a Deductive Argument

\[(p_1 \land p_2 \land \ldots \land p_n) \rightarrow q\]
Valid and Sound Arguments (1 / 2)

**Definition: Valid Argument**

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**Example(s):**

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Valid and Sound Arguments (2 / 2)

**Example(s):**

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**Definition: Sound Argument**

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Some Rules of Inference (1 / 2)

Learn these!

1. Addition

2. Simplification

3. Conjunction

4. Modus Ponens

Some Rules of Inference (2 / 2)

Learn these, too!

5. Modus Tollens

6. Hypothetical Syllogism

7. Disjunctive Syllogism

8. Resolution
Examples of Valid Arguments (1 / 4)

#1: You accidently drop a pen. You know that the pen will fall if it is dropped. How do you know that the pen will fall?

Examples of Valid Arguments (2 / 4)

#2: If 191 is divisible by 7, then $191^2$ is divisible by 49.  

191 is divisible by 7, so $191^2$ is divisible by 49.  

Is this argument valid?
Examples of Valid Arguments (3 / 4)

#3: If you email me a love note, I'll send you flowers. If you don't,
I'll study Discrete Math. If I study Discrete Math, I'll do well on the quiz.
Can we conclude that, if I don't send you flowers, I'll do well on the quiz?

#3: (cont.)

\[ \begin{align*}
  p & : \text{You email me a love note} \\
  q & : \text{I send you flowers} \\
  r & : \text{I study Discrete Math} \\
  s & : \text{I do well on the quiz}
\end{align*} \]

\[ \begin{align*}
  p \rightarrow q \\
  p \rightarrow r \\
  r \rightarrow s
\end{align*} \]

\[ \therefore \quad \overline{q} \rightarrow s \quad ??? \]
Rules of Inference for Predicates (1 / 2)

Four common rules:

1. Universal Instantiation
\[ \forall x \, P(x), \, x \in D \, / \, \therefore P(d) \text{ if } d \in D \]

2. Universal Generalization
\[ P(d) \text{ for any } d \in D \, / \, \therefore \forall x \, P(x), \, x \in D \]

3. Existential Instantiation
\[ \exists x \, P(x), \, x \in D \, / \, \therefore P(d) \text{ for some } d \in D \]

4. Existential Generalization
\[ P(d) \text{ for some } d \in D \, / \, \therefore \exists x \, P(x), \, x \in D \]

Rules of Inference for Predicates (2 / 2)

Example(s):
Definition: Fallacy

Three classic types:

1. Affirming the Conclusion (or . . . Consequent)

2. Denying the Hypothesis (or . . . Antecedent)

3. Begging the Question (a.k.a. Circular Reasoning)
Fallacies for Fun

1. Fallacy of Interrogation

2. ‘No True Scotsman’ Fallacy

Extra Slides

The remaining slides in this topic are some that I no longer cover in class. I won’t ask about them on a quiz or an exam, but they could be referenced on a homework or in SIs.
Specious Reasoning: The Bear Patrol (1 / 3)

Homer: Ah, not a bear in sight. The Bear Patrol must be working like a charm!

Lisa: That’s specious reasoning, Dad. [...] By your logic, I could claim that this rock keeps tigers away!

Homer: Oh . . . and how does it work?

Lisa: It doesn’t work. [...] It’s just a stupid rock. [...] But I don’t see any tigers around here, do you?

Homer: Lisa, I want to buy your rock.

From: The Simpsons, “Much Apu About Nothing”
(Season 7, Episode 151, Production Code 3F20)

Specious Reasoning: The Bear Patrol (2 / 3)

Definition: Specious Reasoning
An unsupported or improperly constructed argument. (That is, an unsound or invalid argument.)

Question: Where is the error in Homer’s logic?

\( b: \) There are bears in Springfield
\( w: \) The Bear Patrol is working

First issue: Which of these is Homer’s argument?

\[
\begin{align*}
(1) & \quad \neg b \quad \text{(Given)} \\
(2) & \quad \therefore w \quad \text{???)} \\
(1) & \quad w \quad \text{(Given)} \\
(2) & \quad \therefore \neg b \quad \text{???)}
\end{align*}
\]

The first seems most reasonable in context.
Question: Where is the error in Homer’s logic? (cont.)

Next, what is the missing piece of Homer’s argument?

(1) \( \neg b \)
(2) \( \neg b \rightarrow w \) ← this is what we’re trying to show!

(3) \( \therefore w \) (1, 2, Modus Ponens)

OK, then, how about . . .

(1) \( \neg b \)
(2) \( w \rightarrow \neg b \) ← might sound good, but . . .

(3) \( \therefore w \) (1, 2, um . . . Abracadabra?)

(The second form of Homer’s argument fails similarly.)