Additional Set Concepts

Set Concepts Already Covered

You may wish to review these basic set concepts, previously covered in the Math Review appendix, before starting this topic:

- Properties of sets (e.g., duplicate members are not allowed)
- Set notation (membership, set builder notation, etc.)
- Operators (union, intersection, difference, complement, cardinality)
- Venn diagrams
Why Are We Studying Sets?

(Sets are trivial . . . aren’t they?)

Sets are foundational in many areas of Computer Science.

E.g.:

Subsets

Definition: Subset

Definition: Proper Subset

Example(s):
Set Equality

**Definition: Set Equality**

**Example(s):**

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Power Sets

**Definition: Power Set**

**Example(s):**
Generalized Forms of $\cup$ and $\cap$

Remember summation and product notations? E.g.:

$$\sum_{n=0}^{9} (2n + 1)$$

Similar notation is used to generalize the union and intersection operators.

Assuming that $A_1 \ldots A_m$ and $B_1 \ldots B_n$ are sets, then:

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Two More Set Properties

**Definition: Disjoint**

**Definition: Partition**

**Example(s):**
Examples of Set Identities

Look familiar?

**Associativity**

- \((A \cap B) \cap C = A \cap (B \cap C)\)
- \((A \cup B) \cup C = A \cup (B \cup C)\)

**Commutativity**

- \(A \cap B = B \cap A\)
- \(A \cup B = B \cup A\)

**Distributivity**

- \(A \cap (B \cup C) = (A \cap B) \cup (A \cap C)\)
- \(A \cup (B \cap C) = (A \cup B) \cap (A \cup C)\)

**De Morgan**

- \(A \cup B = \overline{A} \cap \overline{B}\)
- \(A \cap B = \overline{A} \cup \overline{B}\)

**Note:** As with logical identities, you need not memorize set identities.

Expressing Set Operations in Logic

We’ve seen the first two already.

- \(X \subseteq Y \equiv \forall z (z \in X \to z \in Y)\)
- \(X \subset Y \equiv \forall z (z \in X \to z \in Y) \land \exists w (w \notin X \land w \in Y)\)

For those that return sets, Set Builder notation is a good choice:
To prove that set expressions $S$ and $T$ are equal, we can:

1. Prove that $S \subseteq T$ and $T \subseteq S$, or
2. Convert the equality to logic, prove it, and convert back

Example(s):

Conjecture: $S \cup \mathcal{U} = \mathcal{U}$
Conjecture: $S \cup U = U$
Final Set Operator: Cartesian Product (1 / 2)

**Definition:** Ordered Pair

Example(s):

Final Set Operator: Cartesian Product (2 / 2)

**Definition:** Cartesian Product

Example(s):

Notes: