Background

Having collections of data: Good.

Knowing the connections between collections: Better!

Example(s):
Relations (1 / 2)

**Definition:** (Binary) Relation

Example(s):

Relations (2 / 2)

**Definition:** Related

Example(s):
Example #1: Presidents–Parties

Recall: 
\[ A = \{ \text{Kennedy, Johnson, Nixon, Carter, Reagan} \} \]
\[ B = \{ \text{Dem, Rep} \} \]
\[ R = \{ (\text{Kennedy, Dem}), (\text{Johnson, Dem}), (\text{Nixon, Rep}), (\text{Carter, Dem}), (\text{Reagan, Rep}) \} \]

Kennedy •

Johnson •

Nixon •

Carter •

Reagan •

Democratic

Republican

Example #2: \( x \% y = 0, x \neq y \)

Recall: 
\[ H = \{ 1, 2, 3, 4, 5, 6 \} \]
\[ R = \{ (2, 1), (3, 1), (4, 1), (5, 1), (6, 1), (4, 2), (6, 2), (6, 3) \} \]
Properties of Relations: Reflexivity

Definition: Reflexivity

Example(s):

Properties of Relations: Symmetry (1 / 2)

Definition: Symmetry

Example(s):
Properties of Relations: Symmetry (2 / 2)

Example(s): Graph Representations & Symmetry

Properties of Relations: Antisymmetry (1 / 2)

Definition: Antisymmetry

Example(s):
Properties of Relations: Antisymmetry (2 / 2)

Example(s): Graph Representations & Antisymmetry

Properties of Relations: Transitivity (1 / 2)

Definition: Transitivity

Example(s):
Relational Composition Examples (1 / 4)

Three examples of creating relations from relations.

Example #1: Set Operators
Relational Composition Examples (2 / 4)

Example #2: Swapping content of ordered pairs

Definition: Inverse

Example #3: Composites

Definition: Composite

Example(s):
Definition: Complement

Matrix Representation of Relations (1 / 4)

(Assumption: Relations are on just one set.)

The 0-1 matrix representation of relation $R$ on set $A$ is $|A| \times |A|$, with both dimensions labeled identically. When $(a, b) \in R$, then $\text{matrix}[a][b]=1$. Else, $\text{matrix}[a][b]=0$. 

Example(s):
Observation #1: Detecting Reflexivity

⇒ A relation is reflexive when its corresponding matrix representation has all 1’s along the main diagonal

Example(s):

Observation #2: Detecting Symmetry

⇒ Let matrix $M$ represent relation $R$. $R$ is symmetric when $m_{ij} = 1$ iff $m_{ji} = 1$ is true

Example(s):
Observation #3: Detecting Transitivity

\[ \Rightarrow \text{Let matrix } M \text{ represent relation } R. \text{ } R \text{ is transitive when } M^2 \text{ (or } M^{[2]} \text{) and } M \text{ contain the same quantity of zero elements.} \]

Example(s):


Equivalence Relations (1 / 4)

You may have already implemented one in Java...

Definition: Equivalence Relation
So . . . why are these called equivalence relations?

Recall:

\[ R = \{ (0, 0), \\
(1, 1), (1, -1), (-1, 1), (-1, -1), \\
(2, 2), (2, -2), (-2, 2), (-2, -2) \} \]
Definition: Equivalence Class

Example(s):

Partial Orders (1 / 3)

Consider scheduling the construction of a house.

Definition: Reflexive (a.k.a. Weak) Partial Order
Partial Orders (2 / 3)

Example(s):

Partial Orders (3 / 3)

**Definition: Irreflexivity (of Relations)**

**Definition: Irreflexive (a.k.a. Strict) Partial Order**
Total Orders (1 / 2)

Definition: Comparable

Definition: Total Order

Total Orders (2 / 2)

Example(s):