Background

Having collections of data: Good.

Knowing the connections between collections: Better!

Example(s):
Definition: (Binary) Relation

Example(s):

Definition: Related

Example(s):
Example #1: Presidents–Parties

Recall: \( A = \{ \text{Kennedy, Johnson, Nixon, Carter, Reagan} \} \)
\( B = \{ \text{Dem, Rep} \} \)
\( R = \{ (\text{Kennedy, Dem}), (\text{Johnson, Dem}), (\text{Nixon, Rep}), (\text{Carter, Dem}), (\text{Reagan, Rep}) \} \)

Example #2: \( x \% y = 0, x \neq y \)

Recall: \( H = \{ 1, 2, 3, 4, 5, 6 \} \)
\( R = \{ (2, 1), (3, 1), (4, 1), (5, 1), (6, 1), (4, 2), (6, 2), (6, 3) \} \)

\begin{align*}
1 & \quad 2 \\
6 & \quad 3 \\
5 & \quad 4
\end{align*}
Properties of Relations: Reflexivity

**Definition: Reflexivity**


**Example(s):**


Properties of Relations: Symmetry (1 / 2)

**Definition: Symmetry**


**Example(s):**


Properties of Relations: Symmetry (2 / 2)

Example(s): Graph Representations & Symmetry

Properties of Relations: Antisymmetry (1 / 2)

Definition: Antisymmetry

Example(s):
Properties of Relations: Antisymmetry (2 / 2)

Example(s): Graph Representations & Antisymmetry

Properties of Relations: Transitivity (1 / 2)

Definition: Transitivity

Example(s):
Properties of Relations: Transitivity (2 / 2)

Example(s):

Relational Composition Examples (1 / 4)

Three examples of creating relations from relations.

Example #1: Set Operators
Relational Composition Examples (2 / 4)

Example #2: Swapping content of ordered pairs

**Definition:** Inverse

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Relational Composition Examples (3 / 4)

Example #3: Composites

**Definition:** Composite

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Example(s):
Relational Composition Examples (4 / 4)

Example #3: Composites (cont.)

Example(s):

Definition: Complement

Matrix Representation of Relations (1 / 4)

(Assumption: Relations are on just one set.)

The 0-1 matrix representation of relation $R$ on set $A$ is $|A| \times |A|$, with both dimensions labeled identically. When $(a, b) \in R$, then $\text{matrix}[a][b]=1$. Else, $\text{matrix}[a][b]=0$. 

Example(s):
Matrix Representation of Relations (2 / 4)

Observation #1: Detecting Reflexivity

⇒ A relation is reflexive when its corresponding matrix representation has all 1’s along the main diagonal.

Example(s):

Matrix Representation of Relations (3 / 4)

Observation #2: Detecting Symmetry

⇒ Let matrix $M$ represent relation $R$. $R$ is symmetric when $m_{ij} = 1$ iff $m_{ji} = 1$ is true.

Example(s):
Observation #3: Detecting Transitivity

⇒ Let matrix $M$ represent relation $R$. $R$ is transitive when the non-zero elements of $M^2$ (or of $M^{[2]}$) are also non-zero in $M$.

Example(s):

Equivalence Relations (1 / 4)

You may have already implemented one in Java...

Definition: Equivalence Relation
So . . . why are these called *equivalence* relations?

Recall:

\[ R = \{(0, 0), (1, 1), (1, -1), (-1, 1), (-1, -1), (2, 2), (2, -2), (-2, 2), (-2, -2)\} \]
Equivalence Relations (4 / 4)

Definition: Equivalence Class

Example(s):

Partial Orders (1 / 3)

Consider scheduling the construction of a house.

Definition: Reflexive (a.k.a. Weak) Partial Order
Example(s):

Definition: Irreflexivity (of Relations)

Definition: Irreflexive (a.k.a. Strict) Partial Order
Total Orders (1 / 2)

Definition: Comparable


Definition: Total Order


Total Orders (2 / 2)

Example(s):