Background

Having collections of data: Good.

Knowing the connections between collections: Better!

Example(s):
Definitions:

1. **Binary Relation**

Example(s):

2. **Related**

Example(s):
Example #1: Presidents–Parties

Recall: $A = \{\text{Kennedy, Johnson, Nixon, Carter, Reagan}\}$
$B = \{\text{Dem, Rep}\}$
$R = \{(\text{Kennedy, Dem}), (\text{Johnson, Dem}), (\text{Nixon, Rep}), (\text{Carter, Dem}), (\text{Reagan, Rep})\}$

Kennedy •
Johnson • Democratic
Nixon •
Carter • Republican
Reagan •

Example #2: $x \% y = 0$, $x \neq y$

Recall: $H = \{1, 2, 3, 4, 5, 6\}$
$R = \{(2, 1), (3, 1), (4, 1), (5, 1), (6, 1), (4, 2), (6, 2), (6, 3)\}$

1 • 2

6 • 3

5 • 4
Properties of Relations: Reflexivity

**Definition: Reflexivity**

Example(s):

Properties of Relations: Symmetry (1 / 2)

**Definition: Symmetry**

Example(s):
Properties of Relations: Symmetry (2 / 2)

Example(s): Graph Representations & Symmetry

Properties of Relations: Antisymmetry (1 / 2)

Definition: Antisymmetry

Example(s):
Properties of Relations: Antisymmetry (2 / 2)

Example(s): Graph Representations & Antisymmetry

Properties of Relations: Transitivity (1 / 2)

Definition: Transitivity

Example(s):
Properties of Relations: Transitivity (2 / 2)

Example(s):

Relational Composition Examples (1 / 4)

Three examples of creating relations from relations.

Example #1: Set Operators
Example #2: Swapping content of ordered pairs

**Definition:** Inverse

Example #3: Composites

**Definition:** Composite

**Example(s):**
Example #3: Composites (cont.)

Example(s):

Definition: Complement

Matrix Representation of Relations (1 / 4)

(Assumption: Relations are on just one set.)

The 0-1 matrix representation of relation $R$ on set $A$ is $|A| \times |A|$, with both dimensions labeled identically. When $(a, b) \in R$, then $\text{matrix}[a][b]=1$. Else, $\text{matrix}[a][b]=0$.

Example(s):
Observation #1: Detecting Reflexivity

⇒ A relation is reflexive when its corresponding matrix representation has all 1’s along the main diagonal

Example(s):

Observation #2: Detecting Symmetry

⇒ Let matrix $M$ represent relation $R$. $R$ is symmetric when $m_{ij} = 1$ iff $m_{ji} = 1$ is true

Example(s):
Matrix Representation of Relations (4 / 4)

Observation #3: Detecting Transitivity

⇒ Let matrix $M$ represent relation $R$. $R$ is transitive when the non-zero elements of $M^2$ (or of $M^{[2]}$) are also non-zero in $M$.

Example(s):

Equivalence Relations (1 / 4)

You may have already implemented one in Java...

Definition: Equivalence Relation
So . . . why are these called *equivalence* relations?

Recall:

\[ R = \{ (0, 0), (1, 1), (1, -1), (-1, -1), (1, 1), (2, 2), (2, -2), (-2, 2), (-2, -2) \} \]
Equivalence Relations (4 / 4)

**Definition: Equivalence Class**

Example(s):

Partial Orders (1 / 3)

Consider scheduling the construction of a house.

**Definition: Reflexive (a.k.a. Weak) Partial Order**
Partial Orders (2 / 3)

Example(s):

Partial Orders (3 / 3)

**Definition: Irreflexivity (of Relations)**

Definition: Irreflexive (a.k.a. Strict) Partial Order
Total Orders (1 / 2)

**Definition: Comparable**

**Definition: Total Order**

Total Orders (2 / 2)

**Example(s):**