Background

Having collections of data: Good.

Knowing the connections between collections: Better!

Example(s):
Relations (1 / 2)

Definition: (Binary) Relation

Example(s):

Relations (2 / 2)

Definition: Related

Example(s):
Graph Representations of Relations (1 / 2)

Example #1: Presidents–Parties

Recall: 
\[ A = \{\text{Kennedy, Johnson, Nixon, Carter, Reagan}\} \]
\[ B = \{\text{Dem, Rep}\} \]
\[ R = \{(\text{Kennedy, Dem}), (\text{Johnson, Dem}), (\text{Nixon, Rep}), (\text{Carter, Dem}), (\text{Reagan, Rep})\} \]

- Kennedy
- Johnson
- Nixon
- Carter
- Reagan

- Democratic
- Republican

Graph Representations of Relations (2 / 2)

Example #2: \( x \% y = 0, x \neq y \)

Recall: 
\[ H = \{1, 2, 3, 4, 5, 6\} \]
\[ R = \{(2, 1), (3, 1), (4, 1), (5, 1), (6, 1), (4, 2), (6, 2), (6, 3)\} \]
Properties of Relations: Reflexivity

Definition: Reflexivity

Example(s):

Properties of Relations: Symmetry (1 / 2)

Definition: Symmetry

Example(s):
Properties of Relations: Symmetry (2 / 2)

Example(s): Graph Representations & Symmetry

Properties of Relations: Antisymmetry (1 / 2)

Definition: Antisymmetry

Example(s):
Properties of Relations: Antisymmetry (2 / 2)

Example(s): Graph Representations & Antisymmetry

Properties of Relations: Transitivity (1 / 2)

Definition: Transitivity

Example(s):
Example(s): 

Relational Composition Examples (1 / 4)
Three examples of creating relations from relations.
Example #1: Set Operators
Example #2: Swapping content of ordered pairs

**Definition: Inverse**

Example #3: Composites

**Definition: Composite**

Example(s):
Example #3: Composites (cont.)

Example(s):

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\text{Definition: Complement}
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Matrix Representation of Relations (1 / 4)

(Assumption: Relations are on just one set.)

The 0-1 matrix representation of relation \( R \) on set \( A \) is \(|A| \times |A|\), with both dimensions labeled identically. When \((a, b) \in R\), then \(\text{matrix}[a][b]=1\). Else, \(\text{matrix}[a][b]=0\).

Example(s):
Observation #1: Detecting Reflexivity

⇒ A relation is reflexive when its corresponding matrix representation has all 1’s along the main diagonal

Example(s):

Observation #2: Detecting Symmetry

⇒ Let matrix $M$ represent relation $R$. $R$ is symmetric when $m_{ij} = 1$ iff $m_{ji} = 1$ is true

Example(s):
Observation #3: Detecting Transitivity

⇒ Let matrix $M$ represent relation $R$. $R$ is transitive when the non-zero elements of $M^2$ (or of $M^2$) are also non-zero in $M$.

Example(s):

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Equivalence Relations (1 / 4)

You may have already implemented one in Java...

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Definition: Equivalence Relation
Equivalence Relations (3 / 4)

So . . . why are these called *equivalence* relations?

Recall:

\[ R = \{ (0, 0), (1, 1), (1, -1), (-1, 1), (-1, -1), (2, 2), (2, -2), (-2, 2), (-2, -2) \} \]
Equivalence Relations (4 / 4)

**Definition:** Equivalence Class

Example(s):

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Partial Orders (1 / 3)

Consider scheduling the construction of a house.

**Definition:** Reflexive (a.k.a. Weak) Partial Order

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Partial Orders (2 / 3)

Example(s):

Definition: Irreflexivity (of Relations)

Definition: Irreflexive (a.k.a. Strict) Partial Order
Total Orders (1 / 2)

**Definition: Comparable**

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**Definition: Total Order**

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Total Orders (2 / 2)

**Example(s):**