Background

Having collections of data: Good.

Knowing the connections between collections: Better!

Example(s):
Relations (1 / 2)

**Definition:** (Binary) Relation

Example(s):

Relations (2 / 2)

**Definition:** Related

Example(s):
Graph Representations of Relations (1 / 2)

Example #1: Presidents–Parties

Recall: \( A = \{ \text{Kennedy}, \text{Johnson}, \text{Nixon}, \text{Carter}, \text{Reagan} \} \)
\( B = \{ \text{Dem}, \text{Rep} \} \)
\( R = \{ (\text{Kennedy}, \text{Dem}), (\text{Johnson}, \text{Dem}), (\text{Nixon}, \text{Rep}), (\text{Carter}, \text{Dem}), (\text{Reagan}, \text{Rep}) \} \)

Graph Representations of Relations (2 / 2)

Example #2: \( x \% y = 0, x \neq y \)

Recall: \( H = \{ 1, 2, 3, 4, 5, 6 \} \)
\( R = \{ (2, 1), (3, 1), (4, 1), (5, 1), (6, 1), (4, 2), (6, 2), (6, 3) \} \)
Properties of Relations: Reflexivity

Definition: Reflexivity

Example(s):

Properties of Relations: Symmetry (1 / 2)

Definition: Symmetry

Example(s):
Properties of Relations: Symmetry (2 / 2)

Example(s): Graph Representations & Symmetry

Properties of Relations: Antisymmetry (1 / 2)

Definition: Antisymmetry

Example(s):
Properties of Relations: Antisymmetry (2 / 2)

Example(s): Graph Representations & Antisymmetry

Properties of Relations: Transitivity (1 / 2)

Definition: Transitivity

Example(s):
Properties of Relations: Transitivity (2 / 2)

Example(s):

Relational Composition Examples (1 / 4)

Three examples of creating relations from relations.

Example #1: Set Operators
Relational Composition Examples (2 / 4)

Example #2: Swapping content of ordered pairs

**Definition:** Inverse

Example(s):

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Relational Composition Examples (3 / 4)

Example #3: Composites

**Definition:** Composite

Example(s):

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Example #3: Composites (cont.)

Example(s):

Definition: Complement

Matrix Representation of Relations (1 / 4)

(Assumption: Relations are on just one set.)

The 0-1 matrix representation of relation $R$ on set $A$ is $|A| \times |A|$, with both dimensions labeled identically. When $(a, b) \in R$, then $\text{matrix}[a][b]=1$. Else, $\text{matrix}[a][b]=0$.  

Example(s):
Observation #1: Detecting Reflexivity
⇒ A relation is reflexive when its corresponding matrix representation has all 1’s along the main diagonal

Example(s):

Observation #2: Detecting Symmetry
⇒ Let matrix $M$ represent relation $R$. $R$ is symmetric when $m_{ij} = 1$ iff $m_{ji} = 1$ is true

Example(s):
Observation #3: Detecting Transitivity

⇒ Let matrix $M$ represent relation $R$. $R$ is transitive when $M^2$ (or $M^{[2]}$) and $M$ contain the same quantity of zero elements.

Example(s):

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Equivalence Relations (1 / 4)

You may have already implemented one in Java...

Definition: Equivalence Relation
So . . . why are these called *equivalence* relations?

Recall:

\[ R = \{ (0, 0), (1, 1), (1, -1), (-1, 1), (-1, -1), (2, 2), (2, -2), (-2, 2), (-2, -2) \} \]
Equivalence Relations (4 / 4)

Definition: Equivalence Class

Example(s):

Partial Orders (1 / 3)

Consider scheduling the construction of a house.

Definition: Reflexive (a.k.a. Weak) Partial Order
Partial Orders (2 / 3)

Example(s):

Partial Orders (3 / 3)

Definition: Irreflexivity (of Relations)

Definition: Irreflexive (a.k.a. Strict) Partial Order
Total Orders (1 / 2)

**Definition: Comparable**


**Definition: Total Order**


Total Orders (2 / 2)

**Example(s):**