Functions as Relations (1/2)

Consider: \( f(x) = x + 1, \ x \in \mathbb{Z} \)

**Definition: Function**
Functions as Relations (2 / 2)

Example(s):

Function Terms (1 / 2)

Let \( f : X \rightarrow Y \) be a function. \( f(n) = p \ [ (n, p) \in f ] \).

- \( X \) is the ________ of \( f \)
- \( Y \) is the ________ of \( f \)
- \( f \) ________ \( X \) to \( Y \)
- \( p \) is the ________ of \( n \)
- \( n \) is the ________ of \( p \)
- \( f \)’s ________ is the set of all images of \( X \)’s elements

Note: A function’s range need not equal its codomain.
Example(s):

\[ g = \{ (a, b) \mid b = a/2 \}, \quad a \in \{0, 2, 4, 8\}, \]
\[ b \in \{0, 1, 2, 3, 4, 5\} \]
Two Functions You Need To Know (1 / 4)

1. Floor \( \lfloor x \rfloor \)

**Definition:** Floor Function

**Example(s):**
Two Functions You Need To Know (2 / 4)

1. Floor (\([x]\)) (cont.)

Using Floor for Rounding to the Nearest Integer

Two Functions You Need To Know (3 / 4)

2. Ceiling (\([x]\))

Definition: Ceiling Function

Example(s):
Two Functions You Need To Know (4 / 4)

2. Ceiling (⌈x⌉) (cont.)

Example(s):

Example: Type A UPC Code Check Digits

The check digit equals the image of this function:

\[ s = \text{Sum of digits in positions 1, 3, 5, 7, 9, & 11} \]
\[ t = \text{Sum of digits in positions 2, 4, 6, 8, & 10} \]
\[ u = 3s + t; \text{ the check digit is } (10 - u \mod 10) \mod 10. \]

Using the above sample:

\[ s = 39, \ t = 24, \text{ and } u = 3(39) + 24 = 141. \]

The check digit = \((10 - 141 \mod 10) \mod 10 = 9.\)
Important Distinction: *Continuous* vs. *Discontinuous* Functions

Consider: \( f = \{(x, x + 1) \mid x \in \ldots\} \)

How should the graph of our long-distance calling plan function look?

\[
\text{Cost}(\text{length}) = \begin{cases} 
50 \text{ cents} & \text{if length} \leq 10 \text{ minutes} \\
50 + 5 \cdot \lceil \text{length} - 10 \rceil \text{ cents} & \text{Otherwise}
\end{cases}
\]
Categories of Functions: Injective

**Definition:** Injective Functions (a.k.a. One-to-one)

Example(s):

Categories of Functions: Surjective

**Definition:** Surjective Functions (a.k.a. Onto)

Example(s):
Categories of Functions: Bijective

**Definition**: **Bijective Functions** (a.k.a. One-to-one Correspondence)

**Example(s):**

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Odds and Ends

**Definition**: **Functional Composition**

Let $f : Y \to Z$ and $g : X \to Y$. The composition of $f$ and $g$, denoted $f \circ g$, is the function $h = f(g(x))$, where $h : X \to Z$.

**Definition**: **Inverse Functions**
Definition: Binary Functions

Example(s):