Sequences and Strings

Sequences

Definition: Sequence [1st Attempt]

Notation:

Example(s):
Rules

Recall:

$$\sum_{i=1}^{n} 2i$$

Example(s):

More Notation:

Sequences and Functions

Definition: Sequence [Final Version]

Example(s):
Arithmetic and Geometric Sequences

**Definition:** Arithmetic Sequence (a.k.a. Arithmetic Progression)

... 

**Definition:** Geometric Sequence (a.k.a. Geometric Progression)

... 

**Example(s):**

... 

Arithmetic Series

The sum of the terms of an arithmetic sequence (a.k.a. arithmetic series):

\[ s_n = a_1 + \ldots + a_n = \frac{1}{2}n(a_1 + a_n) \]

Here’s why: First, note that \( a_n = a_1 + (n - 1)d. \)

Next, here are two expressions for \( s_n \):

\[ s_n = a_1 + (a_1 + d) + (a_1 + 2d) + \ldots + (a_1 + (n - 1)d) \]
\[ s_n = (a_n - (n - 1)d) + (a_n - (n - 2)d) + \ldots + (a_n - d) + a_n \]

Sum these expressions, and the \( d \) terms cancel, leaving:

\[ 2s_n = na_1 + na_n, \text{ or } s_n = \frac{1}{2}n(a_1 + a_n). \]
Increasing Sequences

**Definition: Increasing Sequence**

**Definition: Non-Decreasing Sequence**

**Definition: Strictly Increasing Sequence**

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Decreasing Sequences

**Definition: Decreasing Sequence**

**Definition: Non-Increasing Sequence**

**Definition: Strictly Decreasing Sequence**
Subsequences

**Definition: Subsequence**

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**Example(s):**

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Need to Identify a Sequence?

A great resource for sequences:

The Online Encyclopedia of Integer Sequences

(http://oeis.org/)

Example(s):

Strings (1 / 2)  Somewhat beyond the programming language kind . . .

Definition: String

Example(s):
Strings (2/2)

Notation:

- Lambda ($\lambda$) represents the empty (null) string
- $xy$ means strings $x$ and $y$ are concatenated
- Superscripts denote repetition of concatenation
- $|x|$ represents the length of string $x$
- $A^*$ is the set of strings that can be formed using elements of an alphabet $A$.
  - $A^*$ is an infinite set
  - $\lambda \in A^*$

Set Cardinality Revisited (1/5)

An observation about set cardinality:

**Definition: Finite**
**Set Cardinality Revisited (2 / 5)**

**Definition:** Countably Infinite (a.k.a. Denumerably Infinite)

**Definition:** Countable

**Set Cardinality Revisited (3 / 5)**

**Example(s):**
Set Cardinality Revisited (4 / 5)

**Question:** Are the positive rational numbers countable?

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Set Cardinality Revisited (5 / 5)

**Conjecture:** A pairing function for $\mathbb{R}$ cannot exist.
Now You Can Understand More Cartoons! (1/2)

Background: Elephant jokes became popular form of absurdist humor in the U.S. in the 1960s. For example:

Q: How many elephants can fit in a Jeep?
A: Four – Two in the front and two in the back.

Q: How many bison can fit in a Jeep?
A: None – it’s full of elephants.

Q: How do you know when there are two elephants in your closet?
A: You hear giggling when the door is closed.

Q: How do you know when there are three elephants in your closet?
A: You can’t close the door.

Q: How do you know when there are four elephants in your closet?
A: There’s an empty Jeep in the driveway.

Now You Can Understand More Cartoons! (2/2)

http://www.userfriendly.org/cartoons/archives/05jun/uf008006.gif