Finite Probability

Definition: Probability

- The occurrences of interest are called ________________.
- The set of possible occurrences is the ________________.
- These are finite sets, hence the term *finite* probability.
- The occurrence probability of an interest event:
Please note:  
(a) $\forall e \in S, \ p(e) > 0$  
(b) $\sum_{e \in S} p(e) = 1$

Example(s):

Applications of Counting to Probability (1 / 2)

1. Probability of Winning the Powerball Lottery
2. Principle of Inclusion-Exclusion

Recall: \(|E_1 \cup E_2| = |E_1| + |E_2| - |E_1 \cap E_2|\)

Example(s):

Conditional Probability (1 / 2)

Example(s):
Conditional Probability (2 / 2)

Definition: Conditional Probability

Example(s):

Independence of Events (1 / 3)

Recall: \( p(A|B) = \frac{p(A \cap B)}{p(B)} \)

Definition: Independent

Example(s):
Each drawer of a 3x2 dresser holds either a red or a blue UA T-shirt. One row of drawers has two red shirts, one row has two blue, and one row has one of each. You open one drawer and see a red T-shirt. What is the probability that the shirt in the other drawer in the same row is also red?

One solution approach: Enumerate the possibilities. WLOG:

<table>
<thead>
<tr>
<th>Dresser</th>
<th>Open Drawer Containing</th>
<th>Shirt Color in Other Drawer?</th>
</tr>
</thead>
<tbody>
<tr>
<td>R₁</td>
<td>R₁</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>R₂</td>
<td></td>
</tr>
<tr>
<td>R₂</td>
<td>R₃</td>
<td></td>
</tr>
<tr>
<td>R₃</td>
<td>B</td>
<td></td>
</tr>
</tbody>
</table>

Photo Credit: overstock.com
A more famous, more recent, example:

“Suppose you’re on a game show, and you’re given the choice of three doors:
Behind one door is a car; behind the others, goats. You pick a door, say No. 1,
and the host, who knows what’s behind the doors, opens another door, say No.
3, which has a goat. He then says to you, ‘Do you want to pick door No. 2?’ Is
it to your advantage to switch your choice?”


Reference:

www.marilynvossavant.com/game-show-problem/

Care to Play?

But ... why? Three views:

1. Enumerate the Possibilities
Probabilistic Reasoning (5 / 6)

2. ‘Car / Not Car’

Probabilistic Reasoning (6 / 6)

3. Conditional Probability