Directions

1. *This is an INDIVIDUAL assignment; do your own work! Submitting answers created by other people is NOT doing your own work.*

2. Start early! Getting help is much easier five days before the due date/time than it will be five hours before.

3. These questions are over material covered in both the Rosen text (sections 1.4 and 1.5) and “Kneel Before Zodd” (Chapter 2), both accessible from D2L, as well as in our class meetings.

4. Write complete answers to each of the following questions, in accordance with the given directions. Create your solutions as a PDF document such that each question is on a separate page; all parts of a multi-part question may be on the same page. Show your work, when appropriate, for possible partial credit.

5. If you have questions about any aspect of this assignment, help is available from the class staff via piazza.com and our Zoom office hours.

6. When your answers are ready to be turned in, do so on gradescope.com. The entry code is 74DKZK, should you need it. Be sure to assign pages to problems after you upload your PDF. Need help? Visit https://help.gradescope.com/ and search for “Submitting an Assignment.”

7. **Solutions submitted after the first five minutes of class on the due date will not be accepted.**

Rosen, Section 1.4:

1. (8 points) Express these quantified expressions as conversational English sentences. Assume \(R(x) : x\) is a rabbit, \(H(x) : x\) hops, and the domain for both is “Animals.”

   (a) \(\forall x \ (R(x) \rightarrow H(x))\)
   (b) \(\forall x \ (R(x) \land H(x))\)
   (c) \(\exists x \ (R(x) \rightarrow H(x))\)
   (d) \(\exists x \ (R(x) \land H(x))\)

2. (4 points) Express these English sentences as equivalent quantified logical expressions in terms of \(C(x) : x\) has a cat, \(D(x) : x\) has a dog, \(F(x) : x\) has a ferret, and \(S(x) : x\) is enrolled in CSc 245. All domains are “People.”

   (a) Someone enrolled in CSc 245 has a cat and a ferret, but not a dog.
   (b) No one enrolled in CSc 245 has a cat, a dog, and a ferret.

3. (4 points) Assuming that the domain is \(\mathbb{R}\), what is the evaluation (True or False) of each of these expressions? Provide a brief explanation with each of your answers.

   (a) \(\exists x \ (x^3 = -1)\)
   (b) \(\forall x \ ((-x)^2 = x^2)\)

   (Continued . . .)
4. (4 points) Basic quantifications are just short-cuts for conjunctions and disjunctions. Consider the following quantified expressions on the domain \{-2, -1, 0, 1, 2\}. For each, rewrite it as an equivalent unquantified logical expression using conjunctions, disjunctions, and/or negations.

(a) \(\exists x \, P(x)\)
(b) \(\neg \forall x \, P(x)\)

5. (8 points) This is a multi-step question. For each of these English sentences: (i) Express the sentence as an equivalent quantified logical expression in terms of quantifiers that you define (in keeping with our class rules of ‘one concept per predicate’ and ‘do not hide operators’). (ii) Create the negation of your expression such that the negation is to the right of (‘inside’) the quantifier(s). (iii) Express the result of step (ii) in conversational English.

(a) Every koala can climb.
(b) There’s a pig that can swim and catch fish.

Rosen, Section 1.5:

6. (4 points) Assuming a domain of integers, and \(Q(x, y) : x + y = x − y\), what are the evaluations (True or False) of each of these expressions? Provide a brief justification of your answers.

(a) \(\forall x \, \exists y \, Q(x, y)\)
(b) \(\exists y \, \forall x \, Q(x, y)\)

7. (4 points) Express each of these logical expressions in conversational English, assuming that \(C(x, y) : x\) is enrolled in \(y\), with \(x \in \text{Students}\) and \(y \in \text{Classes}\).

(a) \(\exists x \, C(x, \text{“Math 695”})\)
(b) \(\exists a \, \exists b \, \forall c \, ((a \neq b) \land (C(a, c) \rightarrow C(b, c)))\)

8. (6 points) For each of these quantified expressions, express its negation such that all negations are applied directly to predicates (e.g., \(\neg A(x)\)).

(a) \(\exists z \, \forall y \, \exists x \, T(x, y, z)\)
(b) \(\forall y \, \exists x \, \exists z \, (T(x, y, z) \lor Q(x, y))\)

9. (6 points) Let \(W(x) : x\) has an 8K webcam, \(U(x) : x\) is at UArizona, and \(Z(x, y) : x\) and \(y\) have video-chatted over Zoom, where \(x, y \in \text{People}\). Using some/all of those predicates, as well as quantifiers and logical operators, express each of these sentences as logical expressions.

(a) No one at UArizona has video-chatted over Zoom with Augustus De Morgan.
(b) Exactly one person at UArizona has an 8K webcam. (Follow the “exactly one” approach demonstrated in class and in “Kneel Before Zodd.”)

10. (6 points) Let \(Q(x, y) : x\) has been a contestant on \(y\), where \(x \in \text{Students}\) and \(y \in \text{Game Shows}\). Using \(Q\), quantifiers, and logical operators, express each of these sentences as logical expressions.

(a) Every quiz show has had a student as a contestant.
(b) At least two students have been contestants on Jeopardy!. (Reminder: “At least two” is half of the construction of “exactly two.”)

11. (4 points) Express the sentence “Nancy can fool exactly two people” as a quantified logical expression using \(F(x, y) : x\) can fool \(y\), where \(x, y \in \text{People}\). Follow the “exactly two” approach demonstrated in class and in “Kneel Before Zodd.”

12. (2 points) For real numbers, multiplication distributes over addition. For example, \(5 \cdot (1 + 2) = 5 \cdot 1 + 5 \cdot 2\). Express this version of the distributive law using a quantified expression.