Homework #5
(70 points)

Due Date: March 19th, 2021, at the beginning of class

Directions

1. This is an INDIVIDUAL assignment; do your own work! Submitting answers created by other people is NOT doing your own work.

2. Start early! Getting help is much easier five days before the due date/time than it will be five hours before.

3. These questions are over material covered in both the Rosen text (sections 1.7, 2.1, 2.2, 2.6, and 9.1) and “Kneel Before Zodd” (Chapters 5 through 8), both accessible from D2L, as well as in our class meetings.

4. Write complete answers to each of the following questions, in accordance with the given directions. Create your solutions as a PDF document such that each question is on a separate page; all parts of a multi-part question may be on the same page. Show your work, when appropriate, for possible partial credit.

5. If you have questions about any aspect of this assignment, help is available from the class staff via piazza.com and our Zoom office hours.

6. When your answers are ready to be turned in, do so on gradescope.com. The entry code is 74DKZK, should you need it. Be sure to assign pages to problems after you upload your PDF. Need help? Visit https://help.gradescope.com/ and search for “Submitting an Assignment.”

7. Solutions submitted after the first five minutes of class on the due date will not be accepted.

Rosen, Section 1.7; KBZ, Chapter 5:

1. (6 points) Prove, using a Proof by Contradiction: When 0 – e is an even number, e must also be an even number.

2. (6 points) Prove, using a Proof by Contradiction: At least three of the birthdates of 25 people must occur in the same month. (Note that birthdates of March 2000 and March 2001 occur in the same month.) Hint: Start by rewording the conjecture as in implication, to make it easier to see what is assumed to be true.

Rosen, Section 2.1; KBZ, Chapter 6:

3. (4 points) What is the evaluation of each of the following Cartesian Product expressions, when E = \{a, b, c\}, F = \{x, y\}, and G = \{0, 1\}? (Continued . . . )

(a) G × F
(b) G × F × E

4. (2 points) What do we know about the content of each of the two sets Y and Z when Y × Z = \varnothing?
Rosen, Section 2.2; KBZ, Chapter 6:

5. (6 points) Consider the set expression $A \subseteq (A \cup B)$, where $A$ and $B$ are sets. Prove that this expression is true. (Note that, to do this, you will need half of our first way of proving a set identity.)

6. (12 points) Consider the set identity $A = A \cup A$. Prove that this is true using each of the proof techniques we presented it class. That is:
   
   (a) Construct a proof that establishes both (case 1) $A \subseteq A \cup A$ and (case 2) $A \cup A \subseteq A$.
   
   (b) Construct a proof that converts the identity into set–builder notation, provides the argument, and converts out of set–builder notation.

Rosen, Section 2.6; KBZ, Chapter 7:

7. (2 points) If possible, compute
   \[
   \begin{bmatrix}
   -3 & 9 & -3 & 4 \\
   0 & -2 & -1 & 2 \\
   \end{bmatrix}
   \begin{bmatrix}
   -1 & 0 & 5 & 6 \\
   -4 & -3 & 5 & -2 \\
   \end{bmatrix}
   \]. If this operation is not possible on these matrices, explain why not.

8. (2 points) Assume that $A$ is a $3 \times 4$ matrix, $B$ is a $4 \times 5$ matrix, and $C$ is a $4 \times 4$ matrix. There are 9 possible distinct pairings of these three matrices ($AA$, $AB$, $AC$, $BA$, etc.) Which of those nine pairings can be legally evaluated as matrix multiplications?

9. (4 points) If possible, compute the matrix product
   \[
   \begin{bmatrix}
   0 & -1 \\
   7 & 2 \\
   -4 & -3 \\
   \end{bmatrix}
   \begin{bmatrix}
   4 & -1 & 2 & 3 & 0 \\
   -2 & 0 & 3 & 4 & 1 \\
   \end{bmatrix}
   \]. If this operation is not possible on these matrices, explain why not.

10. (4 points) Assume that $A$ is a $6 \times 8$ matrix, $B$ is a $8 \times 4$ matrix, and $C$ is a $4 \times 10$ matrix. How many total multiplications and additions are required to compute each of the following? (See “Kneel Before Zodd” Chapter 7, section 7.4.2.)

   (a) $(AB)C$

   (b) $A(BC)$

11. (8 points) A diagonal matrix is a square matrix $S$ such that $s_{ij} = 0$ if $i \neq j$.

   (a) True or False: The identity matrix $I_5$ is a diagonal matrix.

   (b) True or False: \[
   \begin{bmatrix}
   0 & 0 \\
   0 & 0 \\
   \end{bmatrix}
   \] is a diagonal matrix.

   (c) Prove that the product of two $n \times n$ diagonal matrices is also a diagonal matrix.

12. (6 points) Let $M = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $N = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Evaluate each of the following zero–one matrix expressions.

   (a) $M$ meet $N$

   (b) $M$ join $N$

   (c) $M \odot N$

   (Continued . . .)
13. (4 points) Consider a 2–unit (x–axis) by 5–unit (y–axis) rectangle whose lower-left corner is at (-1,0) and whose upper-right corner is at (1,5). This is the dashed rectangle in the image below. Construct the combined transformation matrix that adjusts this rectangle to be a 10–unit by 2–unit rectangle whose lower–left corner is at (-5,2) (the solid rectangle). Constructing combined transformation matrices was covered in class and is also covered in “Kneel Before Zodd” in section 7.5. Suggestions: (a) Start by translating one corner to the origin. (b) Test your matrix by verifying that it transforms all four corners correctly.

Rosen, Section 9.1; KBZ, Chapter 8:

14. (4 points) Consider a relation \( R \) that contains the ordered pair \((x, y)\) when \( x \mid y \) is true, where \( x, y \in \{1, 2, 3, 4, 5, 6\} \).

   (a) What is the content of \( R \), as a set of ordered pairs?
   (b) Represent \( R \) as a directed graph (a.k.a. digraph).