Directions

1. **This is an INDIVIDUAL assignment; do your own work! Submitting answers created by other people is NOT doing your own work.**

2. Start early! Getting help is much easier five days before the due date/time than it will be five hours before.

3. These questions are over material covered in both the Rosen text (sections 9.1, 9.3, 9.5, 9.6, and 2.3) and “Kneel Before Zodd” (Chapter 8), both accessible from D2L, as well as in our class meetings.

4. Write complete answers to each of the following questions, in accordance with the given directions. Create your solutions as a PDF document such that each question is on a separate page; all parts of a multi-part question may be on the same page. Show your work, when appropriate, for possible partial credit.

5. If you have questions about any aspect of this assignment, help is available from the class staff via piazza.com and our Zoom office hours.

6. When your answers are ready to be turned in, do so on gradescope.com. The entry code is 74DKZK, should you need it. Be sure to assign pages to problems after you upload your PDF. Need help? Visit https://help.gradescope.com/ and search for “Submitting an Assignment.”

7. **Solutions submitted after the first five minutes of class on the due date will not be accepted.**

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Rosen, Section 9.1; KBZ, Chapter 8:

1. (10 points) For each of the following relations, determine whether or not they are (i) reflexive, (ii) irreflexive, (iii) symmetric, (iv) antisymmetric, and (v) transitive.

   (a) $a$ and $b$ were born on the same day, $a, b \in \text{People}$

   (b) $c = 2d, c, d \in \mathbb{R}$

2. (4 points) Let $B = \{1, 2, 3\}$, $C = \{1, 2, 3, 4\}$. $R = \{(1, 2), (2, 3), (3, 4)\}$ and $S = \{(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (3, 4)\}$ are both relations from $B$ to $C$. Evaluate each of the following set expressions:

   (a) $R \cup S$

   (b) $R - S$

3. (4 points) Evaluate: $\{(2, 1), (3, 1), (3, 2), (4, 2)\} \circ \{(1, 2), (1, 3), (2, 3), (2, 4), (3, 1)\}$.

4. (4 points) Let $R = \{(a, b) \mid a > b\}$ and $S = \{(a, b) \mid a \geq b\}$, where $a, b \in \mathbb{R}$. (Yes, $R$ and $S$ are infinite sets.) Evaluate each of the following expressions:

   (a) $S - R$

   (b) $R \circ S$

   (Continued ...)
Rosen, Section 9.3; KBZ, Chapter 8:

5. (2 points) Create the matrix representation of the relation \{(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)\} on the set \{1, 2, 3, 4\}. Label your rows and columns from 1 through 4, in sequence.

6. (6 points) Shown below are matrices \(M_R\) and \(M_S\) that represent relations \(R\) and \(S\), respectively, which are both on set \(A = \{20, 40, 60\}\):

\[
M_R = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad M_S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}
\]

Construct the matrix representations of the relations that result from each of these expressions:

(a) \(R \cup S\)
(b) \(S \circ R\)

7. (4 points) Draw the digraph representations of the relations represented by the matrices \(M_R\) and \(M_S\) from question 6.

8. (2 points) Assume that \(R\) is a relation on set \(A\). How can the digraph representation of \(R\) be used to construct the digraph representation of \(\overline{R}\) (the complement of \(R\))?

Rosen, Section 9.5; KBZ, Chapter 8:

9. (6 points) For each of the following relations, determine whether or not it is an equivalence relation. If the relation is not an equivalence relation, list all of the necessary properties of equivalence relations that the relation does not possess.

(a) \(\{(a, b) \mid a\) and \(b\) have the same parents\}\), where \(a, b \in \text{People}\).
(b) \(\{(a, b) \mid a\) and \(b\) have met\}\), where \(a, b \in \text{People}\).
(c) \(\{(a, b) \mid \text{False}\}\), where \(a, b \in \text{People}\). (Yes, ‘False.’)

10. (6 points) Consider the lovely digraph representation of a relation shown here:

Is the relation represented by this digraph an equivalence relation? If it is, what are the equivalence classes of each of \(a\), \(b\), \(c\), and \(d\)? If it is not, provide a list of edges that could be added to the digraph to make it an equivalence relation.

Rosen, Section 9.6; KBZ, Chapter 8:

11. (12 points) For each of these relations, (i) is the relation a weak partial order, (ii) is it a strict partial order, and (iii) is it a total order? When an answer is ‘no,’ explain why the relation does not possess that property.

(a) \(\{(0, 0), (1, 1), (1, 2), (2, 2), (3, 1), (3, 3)\}\) on \(\{0, 1, 2, 3\}\)
(b) \(\{(0, 0), (0, 1), (0, 2), (0, 3), (1, 0), (1, 1), (1, 2), (1, 3), (2, 0), (2, 2), (3, 3)\}\) on \(\{0, 1, 2, 3\}\)

(c) \[
\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}
\]

(d) (Continued . . .)
12. (2 points) Consider the relation \(R = \{(a, b) \mid a \text{ divides } b\}\), where \(a, b \in \mathbb{Z}^+\). For each pair of integers provided, determine if the pair is comparable in \(R\).

(a) 6, 9
(b) 7, 7

Rosen, Section 2.3:

13. (4 points) For each of the relations shown below, is the relation a function from \(\mathbb{Z}\) to \(\mathbb{R}\)? If a relation is not a function, explain why it is not a function.

(a) \(A = \{(x, y) \mid y = \sqrt{x^2 + 1}\}\)
(b) \(B = \{(x, y) \mid y = \frac{1}{x^2 - 4}\}\)

14. (4 points) Identify the domain and the range (not codomain!) of each of the following functions.

(a) The function that, when given a string of bits, returns the number of one–bits minus the number of zero–bits. (For example, \(f(10011101)\) returns 2.)

(b) The function that returns the first integer of the given ordered pair of positive integers. (For example, \(f((a, b))\) returns \(a\).)