Directions: Write complete answers to each of the following questions, according to the supplied directions. Show your work, when appropriate, for possible partial credit. This is an individual assignment; do your own work. If you need help, remember that we have Piazza and virtual office hours for just this eventuality. The UGTA office hour schedule is available from Piazza and the online syllabus. Please remember that you can talk to any UGTA during office hours, not just your grading group’s UGTA.

On or before the due date and time, email a PDF of your neatly written (or, better yet, electronically–produced) answers to your group’s UGTAs. Our email addresses are available from the class syllabus. Please start your email’s subject line with “CSC245 Homework #” (without the quotes, and with the homework number in place of the #) to help your UGTA manage their email efficiently. Solutions submitted after 3:35 p.m. MST on the due date will not be accepted. Want to be safe and submit your homework early? Feel free to do so; you can email it to your group’s UGTA at any time before the due date.

Helpful Reminders:

- Recurrence relations are describing sequences of values. A solution to a recurrence relation is a closed-form expression that describes the same sequence.
- Checking that both forms produce the same handful of initial values can give you some confidence that you’ve found the closed-form, but a handful of values does not a proof make!
- The Rosen textbook covers recurrence relations in section 2.4 and in Chapter 8 (especially sections 8.2 and 8.3). Checking the examples and odd-numbered exercises in those sections can help you get a better feel for what you need to do to answer these questions.
- We have Piazza and our Zoom office hours if you need help!

The Binomial Theorem.

1. (2 points) How many terms (after all of the like terms are brought together) are in the expansion of \((d + e)^n\)?

2. (2 points) What are the coefficients of the terms \(f^{17}g^{18}\) and \(f^{18}g^{17}\) in the expansion of \((f + g)^{35}\)?

Recurrence Relation Warm–Up Questions.

3. (4 points) For each recurrence, if it is a LHRRWCC of Degree \(k\), what is \(k\)? If it isn’t a LHRRWCC of degree \(k\), explain why not.
   (a) \(r_n = 3r_{n-1}\), where \(r_2 = 2\).
   (b) \(r_n = r_{n-1} \times r_{n-2} \times r_{n-3}\), where \(r_0 = 1, r_1 = 2, r_2 = 4\).

4. (4 points) Find the first four terms (beyond the given initial conditions) of each of the recurrences in Question 3.
Solving Linear Homogeneous Recurrence Relations With Constant Coefficients (LHRRWCC) of Degree 2.

When asked to solve a LHRRWCC of Degree 2, you are expected to use the four-step solution process presented in the lecture slides to produce a closed-form expression that describes the same sequence as that described by the provided recurrence relation.

5. (12 points) Consider the recurrence relation $e_n = e_{n-2}$ with the initial conditions $e_0 = 0$ and $e_1 = 4$.

(a) Solve the recurrence.
(b) Prove that the sequence produced by your closed-form solution to part (a) is the same as the sequence produced by the given recurrence relation.

Solving “Divide and Conquer” Recurrence Relations.

When asked to solve a “Divide and Conquer” recurrence relation, you are expected to use the “Find the Pattern” technique presented in the lecture slides to produce a closed-form expression that describes the same sequence as that described by the provided recurrence relation.

6. (12 points) Consider the recurrence relation $R(n) = R(n-1) - n$ with the initial condition $R(0) = 6$.

(a) Solve the recurrence.
(b) Prove that the sequence produced by your closed-form solution in part (a) is the same as the sequence produced by the given recurrence relation.

7. (12 points) Consider the recurrence relation $S(n) = 2S(n/2) + 3n$ with the initial condition $S(2) = 1$, and assume that $n$ is a power of 2 that is $\geq 2$.

(a) Solve the recurrence.
(b) Prove that the sequence produced by your closed-form solution in part (a) is the same as the sequence produced by the given recurrence relation.

Neatness!

8. (2 points) You’ll receive these points if your answers are legible and easy to follow.