Directions: Write complete answers to each of the following questions, according to the supplied directions. Show your work, when appropriate, for possible partial credit. This is an individual assignment; do your own work. If you need help, remember that we have Piazza and virtual office hours for just this eventuality. The UGTA office hour schedule is available from Piazza and the online syllabus. Please remember that you can talk to any UGTA during office hours, not just your grading group's UGTA.

On or before the due date and time, email a PDF of your neatly written (or, better yet, electronically–produced) answers to your group's UGTAs. Our email addresses are available from the class syllabus. Please start your email's subject line with “CSC245 Homework #” (without the quotes, and with the homework number in place of the #) to help your UGTA manage their email efficiently. Solutions submitted after 3:35 p.m. MST on the due date will not be accepted. Want to be safe and submit your homework early? Feel free to do so; you can email it to your group's UGTA at any time before the due date.

Helpful Reminders:
• I've created lecture videos and slide collections for these topics. If you aren't in the habit of looking at these, you should be.
• The Rosen textbook covers algorithms in sections 3.1 and 5.4, and probability in section 7.1. Checking the examples and odd-numbered exercises in those sections can help you get a better feel for what you need to do to answer these questions.
• Probability is 98% counting. Use your counting concepts to help you determine $|E|$ and $|S|$. After that, the probability is just their ratio.
• We have Piazza and our Zoom office hours if you need help!

Algorithms.

NOTE: Pseudocode was demonstrated in the lecture slides, and is also discussed in Appendix 3 of the Rosen text. Note that I like the word “subprogram” while Rosen likes “procedure”. Either is fine as the start of the definition of a named block of statements that can accept arguments and return values.

1. (6 points) Consider each of our six desirable algorithm characteristics (input, output, generality, definiteness, correctness, and finiteness) in the context of the following pseudocode algorithm. For each of the six, if the algorithm possesses that characteristic, explain how, and if the algorithm does not, explain why not. Assume that rand() returns a random real value in the range [0..1) (that is, including 0 but not including 1).

```
subprogram flipACoin() returns char
begin
    if rand() < 0.5 then return 'H'
    else return 'T'
end
```

(Continued . . .)
2. (6 points) Consider the problem of finding the largest negative integer in a given list of integers and returning its value. For example, the largest negative integer in \([3, -5, -2, 6]\) is \(-2\). If the list contains no negative integers, zero is returned. Write a recursive algorithm, in pseudocode, that solves this problem.

3. (6 points) Using structural induction, prove that your recursive solution to Question 2 is correct.

Probability.

\[ \text{NOTE: For all questions, express your answers as ‘calculator–ready’ expressions, in terms of our counting concepts } (\binom{n}{r}, P(n, r), n!, \text{ etc.}) \text{ when appropriate. You may evaluate them to numeric answers, too, but we need to see your expressions.} \]


(a) You flip a fair coin four times, generating the sequence HTTH. What is the probability of that result occurring?

(b) What is the probability that flipping a fair coin twice produces a head on one of those flips and a tail on the other flip?

(c) What is the probability that flipping a fair coin four times produces two heads and two tails, in any order?

(d) What is the probability that flipping a fair coin ten times produces five heads and five tails, in any order?

5. (10 points) The popular card game UNO has a deck of 108 cards. Four are ‘Wild’, and four are ‘Wild Draw Four’. The remaining cards are grouped into four suits — Blue, Green, Red, and Yellow — with 25 cards per suit. The cards within each suit are a single zero card, two each of the cards one through nine, two ‘Draw Two’ cards, two ‘Skip’ cards, and two ‘Reverse’ cards.

(a) If you draw one card at random from a complete UNO deck, what is the probability that . . .

   i. . . . it is Yellow?

   ii. . . . it is an even–numbered card? (Note: Named cards (‘Draw’, ‘Wild’, etc.) are not numbered.)

   iii. . . . it is either a Yellow card or a ‘Draw Two’ card?

(b) If you deal a seven–card hand from a complete UNO deck, what is the probability that all seven cards are Red?