Math Review: Qs and As

1 Associative, Commutative, and Distributive Laws

Question: Expand: \((2x + x - 4)y\)
Answer: \((2x + x - 4)y = (3x - 4)y = 3xy - 4y\)

Question: Simplify: \(4(x + 3) - 3(-x - 2)\)
Answer: \(4(x + 3) - 3(-x - 2) = 4x + 12 + 3x + 6 = 7x + 18\)

Question: Does \(4 - (1 \cdot 4)\) equal \((4 - 1) \cdot (4 - 4)\)?

Does this show that subtraction distributes over multiplication?
Answer: \(4 - (1 \cdot 4) = 4 - 4 = 0. (4 - 1) \cdot (4 - 4) = 3 \cdot 0 = 0. All this shows is that it is possible to craft an example that makes it appear that subtraction distributes over multiplication, which it does not.

2 Working with Fractions

Question: Simplify: \(\frac{x - 3}{4} - \frac{2x + 1}{3}\)
Answer: \(\frac{x - 3}{4} - \frac{2x + 1}{3} = \frac{3x - 9}{12} - \frac{8x + 4}{12} = \frac{-5x - 13}{12}\) or \(\frac{5x + 13}{12}\)

Question: Simplify: \(\frac{x + 1}{6} \cdot \frac{3}{x}\)
Answer: \(\frac{x + 1}{6} \cdot \frac{3}{x} = \frac{x + 2}{12x} \cdot \frac{3}{x} = \frac{3x + 6}{12x} = \frac{x + 2}{4x}\)

Question: Simplify: \(\frac{xy}{yz}\)
Answer: \(\frac{xy}{yz} = \frac{x}{y} \cdot \frac{y}{zx} = \frac{x}{xyz} = \frac{y}{z}\)

3 The Modulo Operator

Question: Evaluate: \(14 / 4\) and \(14 \% 4\).
Answer: \(14 / 4 = 3\) (using integer division) and \(14 \% 4 = 2\)

Question: List three integers that are each congruent to 17, modulo 6.
Answer: \(17 \% 6 = 5. Any integer +/- a multiple of 6 from 17 will be congruent to 17 modulo 6. Possible values include 23, 41, 5, and -1.\)

4 Exponents and Logarithms

Question: Evaluate \(2^6, 5^{-3}\), and \(3^25^2\).
Answer: \(2^6 = 64. 5^{-3} = \frac{1}{125}. 3^25^2 = 15^2 = 225\)

Question: Evaluate \(4^{\log_y y}, \log_2(2^x), \log_381,\) and \(\log_5125 - \log_51\).
Answer: \(4^{\log_y y} = y. \log_2(2^x) = x. If y = \log_381, then 3^y = 81, and so y = 4.\)
\(\log_5125 - \log_51 = 3 - 0 = 3\)
Question: Express $\log_3(2^x)$ using $\log_{10}$.

Answer: $\log_3(2^x) = \frac{\log_{10}(2^x)}{\log_{10}3}$

5 Factoring Quadratics

Question: Factor $x^2 - 3x - 4$, $3x^2 - 5x + 2$, and $3x^2 + 12x + 9$.

Answer: $x^2 - 3x - 4 = (x + 1)(x - 4)$
$3x^2 - 5x + 2 = (3x - 2)(x - 1)$
$3x^2 + 12x + 9 = 3(x^2 + 4x + 3) = 3(x + 1)(x + 3)$

6 The Quadratic Formula

Question: Solve: $3x^2 - 10x + 8 = 0$ both with and without using the quadratic formula.

Answer: Without: $3x^2 - 10x + 8 = (3x - 4)(x - 2) = 0$. Thus, either $x = \frac{4}{3}$ or $x = 2$.
With: $a = 3$, $b = -10$, $c = 8$. $x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(3)(8)}}{2(3)} = \frac{10 \pm \sqrt{100 - 96}}{6} = \frac{10 \pm 2}{6}$
Thus, as above, $x = 2$ or $x = \frac{4}{3}$.

7 Laws of Inequalities

Question: True or False: $-2 < -3$? $12 \leq 12$? $1.5 > 1.5$?

Answer: False, True, and False, respectively.

Question: Solve for $x$: $5x + 2 \leq x + 12$ and $-4x \geq 9$.

Answer: $5x + 2 \leq x + 12$, $4x \leq 10$, $x \leq \frac{5}{2}$.
$-4x \geq 9$, $4x \leq -9$, $x \leq -\frac{9}{4}$.

8 Summation and Product Notation

Question: Evaluate: $\sum_{i=-2}^{4} \frac{1}{3}$, $\sum_{i=1}^{5} i + 6$ (careful...), and $\prod_{i=1}^{5} i + 6$.

Answer: In $\sum_{i=-2}^{4} \frac{1}{3}$, the sum of the first five terms is 0 (do you see why?), so: $\sum_{i=-2}^{4} \frac{1}{3} = \sum_{i=3}^{4} \frac{1}{3} = \frac{2}{3} + \frac{1}{3} = \frac{7}{3}$
In the second example, note that $\sum_{i=1}^{5} i + 6 \neq \sum_{i=1}^{5} (i + 6)$. Thus $\sum_{i=1}^{5} i + 6 = (1 + 2 + 3 + 4 + 5) + 6 = 21$.
$\prod_{i=1}^{5} i + 6 = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 + 6 = 126$.

9 Number Systems

Question: Convert $749_{10}$ to binary, octal, and hexadecimal.

Answer: $749_{10} = 1011101101_2 = 1355_8 = 2ED_{16}$

Question: Convert $101111100110_{12}$ to octal, hexadecimal, and decimal.

Answer: $101111100110_{12} = 13715_8 = 17CD_{16} = 6093_{10}$