Review Homework #1

Due Date: Friday, April 3, 2020, no later than 3:30 p.m. MST

(110 points)

Directions

1. This is a homework, not an exam. You may refer to the book, past homeworks, etc.

2. As with all homeworks, this is an individual exercise. Create your own answers. Do not seek and do not accept help from anyone other than members of the class staff. In short: Do your own work!

3. You may write your answers in one of three ways:
   (a) Print out these pages, and neatly hand-write your answers below the questions or in the blanks provided;
   (b) Type your clearly labeled answers into a word processor document; or
   (c) Neatly hand-write your clearly labeled answers on blank paper

4. Write/type your name and group # / TA name on the first page of your answers.

5. Make your answers as precise and concise and to the point as possible, while still answering the questions asked.

6. Show your work, where appropriate, for potential partial credit. Vague, incomplete, illegible, and/or ambiguous answers will not receive full credit.

7. If you have general questions, please post them publicly on Piazza. If you have questions specific to your answers, either post them privately on Piazza, or email them to one of the class staff members.

8. When you are ready to submit your answers, convert them to a SINGLE PDF document and email it to your group’s UGTA:

<table>
<thead>
<tr>
<th>Group #</th>
<th>UGTA Name</th>
<th>Email (@email.arizona.edu)</th>
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<tbody>
<tr>
<td>1</td>
<td>Thomas Ruff</td>
<td>truff</td>
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<tr>
<td>2</td>
<td>Angel Aguayo</td>
<td>aaguayo30</td>
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<tr>
<td>3</td>
<td>Kiara Hernandez</td>
<td>kherandez98</td>
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<tr>
<td>4</td>
<td>Carolina Rivera</td>
<td>carolinarivera</td>
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<td>5</td>
<td>Makayla Worden</td>
<td>makaylaworden</td>
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<tr>
<td>6</td>
<td>Jose Gonzalez</td>
<td>gjose</td>
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<td>7</td>
<td>David Gonzales</td>
<td>davidgonzales</td>
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<td>8</td>
<td>Ash Reed</td>
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9. By submitting answers to this homework, you are affirming that you followed these instructions and abided by the academic integrity policies and codes of conduct of this course, the Department of Computer Science, the College of Science, and the University of Arizona.
1. (8 points) Consider the Prolog line `inbetween(X,Y,Z) :- west(X,Y) , west(Y,Z)`. (It’s from Homework #4).
   (a) Is this line a fact or a rule?
   (b) Why are ‘X’, ‘Y’, and ‘Z’ written in upper-case?
   (c) Rewrite that Prolog line as a Horn Clause.

2. (4 points) What is \( P(\{\emptyset, x, y, z\}) \)?

3. (12 points) For the parts of this question, assume that \( G_i = \{v \mid v > i\} \), \( i, v \in \{0,1,2,\ldots,9,10\} \).
   (a) \( G_6 = \)
   (b) \( |G_0| = \)
   (c) \( G_8 \times G_9 = \)
   (d) \( \bigcup_{j=0}^{5} G_{2j} = \)
   (e) Disprove this conjecture by providing a counter-example:
      \[ \forall n \left( G_n \cap G_{n+1} \neq \emptyset \right), \ n \in \{0,1,\ldots,8,9\} . \]
4. (10 points) Prove, using a proof by contraposition, this conjecture:

If \((x + y) \mod 2 = 0\), then \(x\) and \(y\) are both odd or are both even. Assume that \(x, y \in \mathbb{Z}\).

5. (10 points) Prove again, this time using a proof by contradiction, the conjecture from Question 4.

Prove that $\emptyset \cap \emptyset = \emptyset$ using the “$S \subseteq T$ and $T \subseteq S$” approach, by writing a separate proof of each expression.

(a) Prove: $\emptyset \cap \emptyset \subseteq \emptyset$

(b) Prove: $\emptyset \subseteq \emptyset \cap \emptyset$
7. (10 points) Set proofs, Part 2.

Prove that $\emptyset \cap \emptyset = \emptyset$ using the “convert to logic, prove, and convert back to sets” approach.

8. (12 points) Matrix Operations. Let $M = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$ and $N = \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$.

(a) What must be true about the size of a matrix $N$ in order for $N^T + N$ to be computed?

(b) Evaluate: $(4N)^T$.

(c) Evaluate: $MN$.

(d) Evaluate: $M \odot N$. 
9. (4 points) Consider forming a relation on set \( A \). What is the maximum quantity of ordered pairs that can be in that relation? Briefly explain how you arrived at your answer.

10. (10 points) Consider this relation on the set \( V = \{3, 4, 5, 6\} \): \{(3, 5), (3, 6), (4, 6), (5, 3), (6, 3), (6, 4)\}.

   (a) Describe the content of this relation using set builder notation. We’ve provided part of the answer to get you started; you need to fill in the blank space with the rest of the expression. **HINT**: Which ordered pairs are missing from \( V \times V \)?

\[
\{(x, y) \mid \text{______________________________} \}
\]

(b) For each of the following properties, circle yes if this relation has the property or no if it does not. In addition, when you answer “no,” list all of the ordered pairs that would have to be added to the relation so that it would have the property.

   i. reflexive  yes no

   ii. symmetric yes no

   iii. transitive yes no

11. (5 points) Which of these collections of subsets are partitions of \{0, 2, 4, 6, 8, 10, 12\}? For each, circle the correct answer. For those that are not partitions, briefly explain why they are not.

   (a) \{12, 10\}, \{8, 6, 4\}, \{4, 2\} yes no

   (b) \{0, 2, 4\}, \{8, 10, 12\} yes no

   (c) \{12, 0\}, \{6\}, \{2\}, \{10\}, \{4, 8\} yes no

12. (5 points) Create the composition \( R \circ S \circ T \), where \( R = \{(4, b), (8, t), (8, q), (9, y)\} \), \( S = \{(k, 14), (v, 8), (k, 9), (q, 8)\} \), and \( T = \{((X, c), (III, q), (VI, k), (MMXX, v))\} \). (Yes, the domain of \( T \) is Roman numerals.)