Functional Dependencies

Key distinction between relationships and functional dependencies:

- Relationships are between relations
- Functional Dependencies are between attributes
  (usually within the same relation)
Example(s):

**UA Campus Buildings:**

<table>
<thead>
<tr>
<th>UABUILDINGS</th>
<th>Building#</th>
<th>BuildingName</th>
<th>Address</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>77</td>
<td>Gould-Simpson</td>
<td>1040 E. 4th St.</td>
</tr>
<tr>
<td></td>
<td>88</td>
<td>Biological Sciences West</td>
<td>1041 E. Lowell St.</td>
</tr>
</tbody>
</table>

**Appropriate FDs:**
- Building# $\rightarrow$ BuildingName
- Building# $\rightarrow$ Address

**Inappropriate FDs:**
- 
- 

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**Functional Determination (1 / 3)**

**Definition:** Functional Determination
Consider this relation:

<table>
<thead>
<tr>
<th>Id</th>
<th>Name</th>
<th>DAbbrev</th>
<th>DName</th>
<th>DOffice</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Phil</td>
<td>CSc</td>
<td>Computer Science</td>
<td>G-S 917</td>
</tr>
<tr>
<td>4</td>
<td>Lisa</td>
<td>CSc</td>
<td>Computer Science</td>
<td>G-S 917</td>
</tr>
<tr>
<td>5</td>
<td>Steve</td>
<td>Math</td>
<td>Mathematics</td>
<td>Math 108</td>
</tr>
<tr>
<td>13</td>
<td>Bob</td>
<td>CSc</td>
<td>Computer Science</td>
<td>G-S 917</td>
</tr>
<tr>
<td>14</td>
<td>Pat</td>
<td>Math</td>
<td>Mathematics</td>
<td>Math 108</td>
</tr>
</tbody>
</table>

Appropriate FD examples (due to functional behaviors):

- 
- 
- 

Inappropriate FD Examples (non–functional behaviors):

- 
- 

Consider this relation one more time:

<table>
<thead>
<tr>
<th>Id</th>
<th>Name</th>
<th>DAbbrev</th>
<th>DName</th>
<th>DOffice</th>
</tr>
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<td>Math</td>
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<td>Math 108</td>
</tr>
</tbody>
</table>

Inappropriate FD Examples (non–functional behaviors):

- 
- 


The Utility of Functional Dependencies

Why are functional dependencies important? They:

Closure of Attribute Sets (1 / 3)

**Definition: Closure (of a set of attributes)**
Closure of Attribute Sets (2 / 3)

The Closure Algorithm for Attribute Sets:

Given: \( A \) (the set of attributes we wish to close)
\( F \) (the set of existing FDs)

Returns: \( A^+ \) (the closure of \( A \))

Example(s): Consider the six attributes \( U, V, W, X, Y, Z \) and the FDs \( Z \rightarrow Y X, V \rightarrow XU, Y \rightarrow V, \) and \( XW \rightarrow VU \). Find the closure of \( ZY (= \{ Z, Y \}) \).

<table>
<thead>
<tr>
<th>FD</th>
<th>ZY⁺</th>
<th>temp</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Closure of Attribute Sets (3 / 3)
Identifying Important Functional Dependencies

1. List the ‘easy’ FDs.
   - Some are obvious (e.g., Primary Key → all attrs)
   - Context provides others (e.g., \{Dabbrev\} → \{Dname\})

2.

3.

Closure of Functional Dependencies (1 / 2)

From closures of attribute sets, we move to closures of FDs.

**Definition: Closure (of a set of FDs)**
Example(s):

Consider this odd schema:

<table>
<thead>
<tr>
<th>S#</th>
<th>P#</th>
<th>City</th>
<th>Qty</th>
</tr>
</thead>
</table>

Clearly \( \{S\#, P\#\} \rightarrow \{\text{City}\} \) and \( \{S\#, P\#\} \rightarrow \{\text{Qty}\} \).

Armstrong’s Axioms  
(a.k.a. FD Inference Rules)

Additional Inference Rules (1 / 2)

(Remember: These are not Armstrong's Axioms!)

Additional Inference Rules (2 / 2)

As rules 4-8 aren’t fundamental, we can prove their validity using Armstrong’s Axioms.

Example(s): Prove Union: If \( J \rightarrow K \) and \( J \rightarrow L \), then \( J \rightarrow KL \).
Covers of Functional Dependencies (1 / 2)

Covers are essentially the opposite of closures.

![Diagram showing Closure ("Explosion") and Minimal Cover ("Implosion")](image)

Covers of Functional Dependencies (2 / 2)

How many FDs do we need to maintain? (Answer: A minimal cover!)

**Definition:** Covers (of sets of FDs)

**Definition:** Equivalence (of sets of FDs)
Minimal Sets of FDs

(Remember, we're working toward a minimal cover)

Definition: Minimal Sets (of FDs)

A set of FDs is minimal if all three of the following hold:

1. 
2. 
3. 

Minimal Covers of FDs

With a minimal set of FDs defined, the definition of a minimal cover is easy.

Definition: Minimal Cover (of a set of FDs)
FD Minimal Cover Algorithm (1 / 4)

First, a high–level outline of the algorithm:

[ Given: \( MC \), a set of FDs whose minimal cover we wish to find. ]

1. For each FD \( f \in MC \), put \( f \) in Standard Form
   — That is, minimize \( f \)'s RHS (right–hand side)
2. For each FD \( f \in MC \), minimize \( f \)'s LHS (left–hand side)
3. For each FD \( f \in MC \), delete \( f \) if it is redundant

FD Minimal Cover Algorithm (2 / 4)

And now the details!

1. For each FD \( f \in MC \), put \( f \) in Standard Form.
2. For each FD $f \in MC$, minimize $f$’s LHS, using this algorithm:

3. For each FD $f \in MC$, delete $f$ if it is redundant, using this algorithm:
Example(s):

Attributes: \( E, F, G, H \)
Initially, \( MC = \{ E \rightarrow FG, F \rightarrow G, E \rightarrow F, EF \rightarrow G, EG \rightarrow H \} \)

**Question**: What is a minimal cover of this set of FDs?

1. For each FD \( f \in MC \), put \( f \) in Standard Form.
   
   \( \Rightarrow \) Only \( E \rightarrow FG \) is not in Standard Form

Minimal Cover Example (2 / 4)

Example(s):

2. For each FD \( f \in MC \), minimize \( f \)'s LHS.
   
   \( \Rightarrow \) Two candidates: \( EF \rightarrow G \) and \( EG \rightarrow H \).

(a) Consider \( EF \rightarrow G \):
Example(s):

2. For each FD \( f \in MC \), minimize \( f \)'s LHS (continued).
   
   (b) Consider \( EG \rightarrow H \):

Example(s):

3. For each FD \( f \in MC \), delete \( f \) if it is redundant.
   
   \[ \Rightarrow \text{Consider } E \rightarrow G. \]