Functional Dependencies (1 / 2)

Key distinction between relationships and functional dependencies:

- Relationships are between relations
- Functional Dependencies are between attributes
  (usually within the same relation)
Example(s):

UA Campus Buildings:

<table>
<thead>
<tr>
<th>UABUILDINGS</th>
<th>Building#</th>
<th>BuildingName</th>
<th>Address</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>77</td>
<td>Gould-Simpson</td>
<td>1040 E. 4th St.</td>
</tr>
<tr>
<td></td>
<td>88</td>
<td>Biological Sciences West</td>
<td>1041 E. Lowell St.</td>
</tr>
</tbody>
</table>

Appropriate FDs:
- Building# → BuildingName
- Building# → Address

Inappropriate FDs:
- 
- 

Functional Determination (1 / 3)

Definition: Functional Determination
Consider this relation:

<table>
<thead>
<tr>
<th>Id</th>
<th>Name</th>
<th>DAbbrev</th>
<th>DName</th>
<th>DOffice</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Phil</td>
<td>CSc</td>
<td>Computer Science</td>
<td>G-S 917</td>
</tr>
<tr>
<td>4</td>
<td>Lisa</td>
<td>CSc</td>
<td>Computer Science</td>
<td>G-S 917</td>
</tr>
<tr>
<td>5</td>
<td>Steve</td>
<td>Math</td>
<td>Mathematics</td>
<td>Math 108</td>
</tr>
<tr>
<td>13</td>
<td>Bob</td>
<td>CSc</td>
<td>Computer Science</td>
<td>G-S 917</td>
</tr>
<tr>
<td>14</td>
<td>Pat</td>
<td>Math</td>
<td>Mathematics</td>
<td>Math 108</td>
</tr>
</tbody>
</table>

Appropriate FD examples (due to functional behaviors):

- 
- 
- 

Inappropriate FD Examples (non–functional behaviors): 

- 
-
The Utility of Functional Dependencies

Why are functional dependencies important? They:

Closure of Attribute Sets (1 / 3)

Definition: Closure (of a set of attributes)
Closure of Attribute Sets (2 / 3)

The Closure Algorithm for Attribute Sets:

Given: \( A \) (the set of attributes we wish to close)
\( F \) (the set of existing FDs)

Returns: \( A^+ \) (the closure of \( A \))

Closure of Attribute Sets (3 / 3)

Example(s): Consider the six attributes \( U, V, W, X, Y, Z \) and the FDs \( Z \rightarrow Y \; X \), \( V \rightarrow X \; U \), \( Y \rightarrow V \), and \( X \; W \rightarrow V \; U \). Find the closure of \( ZY (= \{Z, Y\}) \).

<table>
<thead>
<tr>
<th>FD</th>
<th>( ZY^+ )</th>
<th>temp</th>
</tr>
</thead>
</table>
Identifying Important Functional Dependencies

1. List the ‘easy’ FDs.
   - Some are obvious (e.g., Primary Key → all attrs)
   - Context provides others (e.g., \{Dabbrev\} → \{Dname\})

2. Closure of Functional Dependencies (1 / 2)

   From closures of attribute sets, we move to closures of FDs.

   **Definition: Closure (of a set of FDs)**
Example(s):

Consider this odd schema:

| S# | P# | City | Qty |

Clearly $\{S\#, P\#\} \rightarrow \{\text{City}\}$ and $\{S\#, P\#\} \rightarrow \{\text{Qty}\}$.

Armstrong’s Axioms  
(a.k.a. FD Inference Rules)

As rules 4 – 8 aren’t fundamental, we can prove their validity using Armstrong’s Axioms.

Example(s): Prove Union: If $J \rightarrow K$ and $J \rightarrow L$, then $J \rightarrow KL$. 
Covers are essentially the opposite of closures.

How many FDs do we need to maintain? (Answer: A minimal cover!)

**Definition: Covers (of sets of FDs)**

**Definition: Equivalence (of sets of FDs)**
Minimal Sets of FDs

Definition: Minimal Sets (of FDs)

A set of FDs is minimal if all three of the following hold:

- All attributes in the left-hand side of the dependencies are minimal.
- All attributes in the right-hand side of the dependencies are minimal.
- No attributes can be removed from the left-hand side without violating any of the dependencies.

Minimal Covers of FDs

With a minimal set of FDs defined, the definition of a minimal cover is easy.

Definition: Minimal Cover (of a set of FDs)

- The minimal cover contains the minimal set of functional dependencies.
- The minimal cover is the smallest set of dependencies that can yield the same functional dependencies as the original set.
- The minimal cover is not subject to the duplication of functional dependencies.
First, a high–level outline of the algorithm:

[ Given: $MC$, a set of FDs whose minimal cover we wish to find. ]

1. For each FD $f \in MC$, put $f$ in Standard Form
   — That is, minimize $f$’s RHS (right–hand side)
2. For each FD $f \in MC$, minimize $f$’s LHS (left–hand side)
3. For each FD $f \in MC$, delete $f$ if it is redundant

And now the details!

1. For each FD $f \in MC$, put $f$ in Standard Form.
2. For each FD $f \in MC$, minimize $f$’s LHS, using this algorithm:

3. For each FD $f \in MC$, delete $f$ if it is redundant, using this algorithm:
Example(s):

Attributes: $E, F, G, H$

Initially, $MC = \{ E \rightarrow FG, F \rightarrow G, E \rightarrow F, EF \rightarrow G, EG \rightarrow H \}$

**Question:** What is a minimal cover of this set of FDs?

1. For each FD $f \in MC$, put $f$ in Standard Form.
   \[
   \Rightarrow \text{Only } E \rightarrow FG \text{ is not in Standard Form}
   \]

Example(s):

2. For each FD $f \in MC$, minimize $f$'s LHS.
   \[
   \Rightarrow \text{Two candidates: } EF \rightarrow G \text{ and } EG \rightarrow H.
   \]
   (a) Consider $EF \rightarrow G$:
Example(s):

2. For each FD \( f \in MC \), minimize \( f \)'s LHS (continued).
   
   (b) Consider \( EG \rightarrow H \):

Example(s):

3. For each FD \( f \in MC \), delete \( f \) if it is redundant.

   \( \Rightarrow \) Consider \( E \rightarrow G \).