Topic 12:

Functional Dependencies

Functional Dependencies (1 / 2)

Key distinction between relationships and functional dependencies:

- Relationships are between relations
- Functional Dependencies are between attributes (usually within the same relation)

Functional Dependencies (2 / 2)

Example(s):

UA Campus Buildings:

UABUILDINGS	Building#	BuildingName	Address
	77	Gould-Simpson	1040 E. 4th St.
	88	Biological Sciences West	1041 E. Lowell St.

Appropriate FDs:

- Building# $\longrightarrow$ BuildingName
- Building# $\longrightarrow$ Address

Inappropriate FDs:

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Functional Determination (1 / 3)

Definition: Functional Determination

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Functional Determination (2 / 3)

Consider this relation:

Student_Department

Id	Name	DAbbrev	DName	DOffice
2	Phil	CSc	Computer Science	G-S 917
4	Lisa	CSc	Computer Science	G-S 917
5	Steve	Math	Mathematics	Math 108
13	Bob	CSc	Computer Science	G-S 917
14	Pat	Math	Mathematics	Math 108

Appropriate FD examples (due to functional behaviors):

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Functional Determination (3 / 3)

Consider this relation one more time:

Student_Department

Id	Name	DAbbrev	DName	DOffice
2	Phil	CSc	Computer Science	G-S 917
4	Lisa	CSc	Computer Science	G-S 917
5	Steve	Math	Mathematics	Math 108
13	Bob	CSc	Computer Science	G-S 917
14	Pat	Math	Mathematics	Math 108

Inappropriate FD Examples (non–functional behaviors):

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The Utility of Functional Dependencies

Why are functional dependencies important? They:

Closure of Attribute Sets (1 / 3)

Definition: Closure (of a set of attributes)

Closure of Attribute Sets (2 / 3)

The Closure Algorithm for Attribute Sets:

Given:	A	(the set of attributes we wish to close)
	F	(the set of existing FDs)
Returns:	A^+	(the closure of A)

Closure of Attribute Sets (3 / 3)

Example(s): Consider the six attributes U, V, W, X, Y, Z and the FDs $Z \rightarrow YX,\ V \rightarrow XU,\ Y \rightarrow V$, and $XW \rightarrow VU$. Find the closure of $ZY (= \{Z, Y\})$.

FD	ZY^+	temp

Identifying Important Functional Dependencies

1. List the 'easy' FDs.
   - Some are obvious (e.g., Primary Key $\rightarrow$ all attrs)
   - Context provides others (e.g., $\{$Dabbrev$\} \rightarrow \{$Dname$\}$)

2.

3.

Closure of Functional Dependencies (1 / 2)

From closures of attribute sets, we move to closures of FDs.

Definition: Closure (of a set of FDs)

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Closure of Functional Dependencies (2 / 2)

Example(s):

Consider this odd schema:

S#	P#	City	Qty

Clearly $\{S\#, P\#\} \rightarrow \{\text{City}\}$ and $\{S\#, P\#\} \rightarrow \{\text{Qty}\}$.

Armstrong's Axioms (a.k.a. FD Inference Rules)

(Armstrong, W. "Dependency Structures of Data Base Relationships," IFIP Congress, 1974.)

Additional Inference Rules (1 / 2)

(Remember: These are **not** Armstrong's Axioms!)

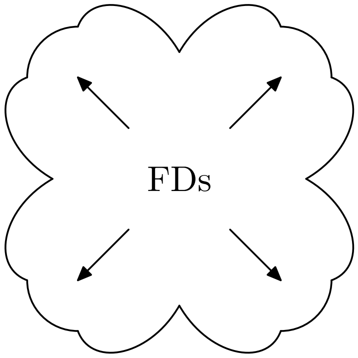
Additional Inference Rules (2 / 2)

As rules 4 – 8 aren't fundamental, we can prove their validity using Armstrong's Axioms.

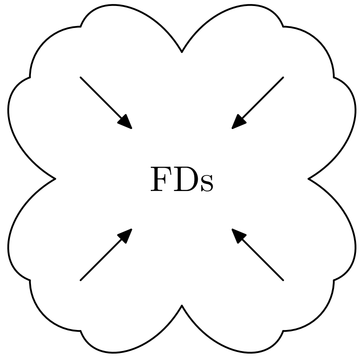
Example(s): Prove Union: If $J \rightarrow K$ and $J \rightarrow L$, then $J \rightarrow KL$.

Covers of Functional Dependencies (1 / 2)

Covers are essentially the opposite of closures.



Closure
("Explosion")



Cover
("Implosion")

Covers of Functional Dependencies (2 / 2)

How many FDs do we need to maintain? (Answer: A *minimal* cover!)

Definition: Covers (of sets of FDs)

. . . .

Definition: Equivalence (of sets of FDs)

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Minimal Sets of FDs (Remember, we're working toward a *minimial* cover)

Definition: Minimal Sets (of FDs)

A set of FDs is minimal if all three of the following hold:

Minimal Covers of FDs

With a minimal set of FDs defined, the definition of a minimal cover is easy.

Definition: Minimal Cover (of a set of FDs)

FD Minimal Cover Algorithm (1 / 4)

First, a high–level outline of the algorithm:

[Given: MC, a set of FDs whose minimal cover we wish to find.]

1. For each FD $f \in MC$, put f in Standard Form
   — That is, minimize f's RHS (right–hand side)
2. For each FD $f \in MC$, minimize f's LHS (left–hand side)
3. For each FD $f \in MC$, delete f if it is redundant

FD Minimal Cover Algorithm (2 / 4)

And now the details!

1. For each FD $f \in MC$, put f in Standard Form.

FD Minimal Cover Algorithm (3 / 4)

2. For each FD $f \in MC$, minimize f's LHS, using this algorithm:

FD Minimal Cover Algorithm (4 / 4)

3. For each FD $f \in MC$, delete f if it is redundant, using this algorithm:

Minimal Cover Example (1 / 4)

Example(s):

Attributes: E, F, G, H

Initially, MC = $\{E \rightarrow FG, F \rightarrow G, E \rightarrow F, EF \rightarrow G, EG \rightarrow H\}$

Question: *What is a minimal cover of this set of FDs?*

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1. For each FD $f \in MC$, put f in Standard Form.

   $\Rightarrow$ Only $E \rightarrow FG$ is not in Standard Form

Minimal Cover Example (2 / 4)

Example(s):

2. For each FD $f \in MC$, minimize f's LHS.

   $\Rightarrow$ Two candidates: $EF \rightarrow G$ and $EG \rightarrow H$.

   (a) Consider $EF \rightarrow G$:

Minimal Cover Example (3 / 4)

Example(s):

2. For each FD $f \in MC$, minimize f's LHS (continued).

   (b) Consider $EG \rightarrow H$. Can we form either $E \rightarrow H$ or $G \rightarrow H$?

Minimal Cover Example (4 / 4)

Example(s):

3. For each FD $f \in MC$, delete f if it is redundant.

   $\Rightarrow$ Consider $E \rightarrow G$.